Abstracts of contributed talks

► PETER B. ANDREWS AND CHAD E. BROWN, *Proving theorems and teaching logic with TPS and ETPS*.
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The Theorem Proving System TPS can be used to construct and check formal proofs interactively, semi-automatically, and automatically. Theorems are expressed in Church's type theory. This includes first-order logic, but in a practical sense it has greater expressive power, and it is particularly well suited to the formalization of mathematics and other disciplines. In automatic mode, TPS first searches for an *expansion proof*, which is a higher-order analogue of a Herbrand expansion, and then transforms this into a proof in natural deduction style.

We show how TPS can prove various theorems automatically or semi-automatically. Some examples of theorems which TPS can prove automatically are:

THM15B: If some iterate of function f has a unique fixed point, then f has a fixed point.

THM136: The transitive closure of a relation is transitive.

THM145: In a complete lattice, every monotone function has a fixed point.

THM531E: A subset of a finite set is finite.

A related program called ETPS contains only interactive facilities, and is used by logic students to construct formal proofs in natural deduction style.

Both TPS and ETPS are available from the web. For more information see [1], [2], and http://gtps.math.cmu.edu/tps.html.

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[1] PETER B. ANDREWS, MATTHEW BISHOP, SUNIL ISSAR, DAN NESMITH, FRANK PFENNING, AND HONGWEI XI, *TPS: A theorem proving system for classical type theory*, ***Journal of Automated Reasoning***, vol. 16 (1996), pp. 321–353.

[2] PETER B. ANDREWS, MATTHEW BISHOP, CHAD E. BROWN, SUNIL ISSAR, FRANK PFENNING, AND HONGWEI XI, *ETPS: A system to help students write formal proofs*, ***Journal of Automated Reasoning***, vol. 32 (2004), to appear.

► CHAD E. BROWN, *Set comprehension in Church's type theory*.
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In order to prove theorems involving sets and functions, one often uses comprehension principles asserting the existence of certain sets. We explore the consequences of restricting which logical constants are allowed in these comprehension principles in the context of Church's type theory.

Church's type theory is a formulation of higher-order logic that allows quantification over sets and functions. In this logic, any set definable by an expression of the language can be proved to exist. This expressive power allows a natural formalization of much of mathematics.

The logical constants we consider are propositional connectives, equality and quantifiers over various types. For example, we consider fragments of Church's type theory which satisfy comprehension with respect to quantifiers over individuals, but not over sets of individuals. There are natural sequent calculi and corresponding semantics for any given signature. In some cases, adding logical constants to a signature does not increase the set of theorems (giving conservation results). In other cases, adding logical constants does increase the set of theorems (giving independence results). We establish these results using models of fragments of type theory.

For example, the usual proof of Cantor's theorem that there is no surjection from a set S onto its power set $\mathcal{P}(S)$ uses a diagonal set whose definition involves a negation. We construct a model showing that this theorem cannot be proven in a fragment of Church's type theory which lacks comprehension principles involving negation. To prove the version of Cantor's theorem that there is no injection from $\mathcal{P}(S)$ into S requires comprehension principles involving quantifiers over sets in $\mathcal{P}(S)$ and equality of objects in S as well as negation.

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► WALTER DEAN, *From Church's thesis to extended Church's thesis*.
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Extended Church's Thesis [ECT] is the claim that any informal algorithm (e.g., Euclid's algorithm, Mergesort) can be analyzed as an instance of a formal model of computation such (e.g., a particular Turing machine) in a manner that preserves its identifying computational properties. ECT is thus a far stronger statement than that traditionally identified as Church's Thesis [CT]: the latter seeks to analyze only the extension of the concept 'function computed by an algorithm' while the former seeks to analyze the intensional properties of individual mathematical procedures.

There is a modern tendency to either misidentify CT as ECT or to consider arguments originally given in favor of CT as having actually established ECT. Gödel, Kreisel, Davis, Sipser and Lewis & Papadimitriou have all made statements to this effect, the latter describing CT as follows:

> [W]e take the Turing machine to be a precise formal equivalent of the intuitive notion of "algorithm": nothing will be considered an algorithm if it cannot be rendered as a Turing machine. The principle that Turing machines are formal versions of algorithms and that no computational procedure will be considered an algorithm unless it can be presented as a Turing machine is known as Church's Thesis . . .

This talk will consider the historical and conceptual passage from CT to ECT. I will first argue that while it is doubtful that ECT would have been accepted during the 1930s, Kleene and Rogers both present traditional arguments for CT as establishing a claim similar to ECT. Next, I will introduce a framework for evaluating versions of ECT given in terms of arbitrary models of computation. Finally, I will argue that even relative to the models recently proposed by Moschovakis and Gurevich, the corresponding versions of ECT are likely to be false.

► ROD DOWNEY AND LIANG YU, *There are no maximal low d.c.e. degrees*.
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In [1], Arslanov, Cooper and Li claimed that for every low L and c.e. set A with $L <_T A$, there exists a c.e. splitting $A_1 \sqcup A_2 = A$ such that $A_1 \oplus L |_T A_2 \oplus L$. As a consequence Arslanov, Cooper and Li observed that there is no maximal low d.c.e. Turing degree. Unfortunately, the proof of Arslanov et al. contains a fatal flaw. We give a direct proof of the latter claim of Arslanov et al. by showing that there is no maximal low d.c.e. degree.

The technique is of some interest, since it would seem one of the few results which use lowness via the "Robinson trick" outside of the c.e. degrees. We also prove that for any low d.c.e. degree, there is a low d.c.e. degree above it and so there are infinitely many d.c.e. degrees above it. But we do not know whether there is a d.c.e. splitting of $0'$ above it.

[1] M. Arslanov, S. B. Cooper and Angsheng Li, *There is no low maximal d.c.e. degree*, ***Mathematical Logic Quarterly***, vol. 46 (2000), pp. 409–416.

► SERGIO FRATARCANGELI, *Elimination of imaginaries in generic expansions of o-minimal theories*.
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Let T be a complete first-order L-theory with quantifier elimination and the uniform finiteness property. Let P be a unary predicate not contained in L. Pillay and Chatzidakis showed that T has a model companion, T_P, in the language $L \cup P$. In addition, they proved that if T is stable and eliminates imaginaries, then T_P also eliminates imaginaries. In this talk, we'll sketch a proof that this result holds when "stable" is replaced by "o-minimal".

► THOMAS KENT, *Results on non-splitting Σ_2^0 enumeration degrees*.
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We will define enumeration reducibility and give a brief history of the enumeration degrees. We show that these non-splitting Σ_2^0 degrees are downward dense in the Δ_2^0 e-degrees and show the existence of a properly Σ_2^0 e-degree. We will examine current progress towards determining the decidability of the Π_2 theory of the Σ_2^0 e-degrees.

► ROMAN KUZNETS, *On decidability of the logic of proofs with arbitrary constant specifications*.
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Logic of Proofs $\mathcal{LP}$ introduced by Artemov gave an exact intended semantics for Gödel's logic of provability $S4$ [1], [2]. This Logic of Proofs considers statements of the form $t\colon F$ where a proof term t (called proof polynomial) denotes a proof for F. Proof polynomials are built from variables and proof constants c which stand for proofs of axioms of the theory. Logic of Proofs has a natural arithmetical semantics where $t\colon F$ is interpreted as a formal arithmetical statement "t is a proof of F in $\mathcal{PA}$." Proof constants are specified by accepting as postulates constant specifications $\mathcal{CS}$ which are sets of formulas of sort $\{c_1\colon A_1, c_2\colon A_2, \dots\}$ where c_i is a proof constant and A_i an axiom. A theory $\mathcal{LP}_{\mathcal{CS}}$ is a theory with constant specification $\mathcal{CS}$. Logic $\mathcal{LP}_0$ corresponding to the empty $\mathcal{CS}$ was shown to be decidable in [1]. Mkrtychev in [3] has shown that if $\mathcal{CS}$ contains only a finite number of axiom schemes for each constant then $\mathcal{LP}_{\mathcal{CS}}$ is decidable. We show that those results do not extend to all decidable constant specifications.

Theorem 1. *There is a decidable constant specification $\mathcal{CS}$ such that the logic $\mathcal{LP}_{\mathcal{CS}}$ is undecidable.*

Moreover, it can be shown that even a decidable constant specification involving only one constant already may lead to an undecidable theory $\mathcal{LP}_{\mathcal{CS}}$.

[1] Sergei N. Artemov, *Operational modal logic*, Technical Report MSI95-29, Cornell University, 1995.

[2] ———, *Explicit provability and constructive semantics*, ***The Bulletin of Symbolic Logic***, vol. 7 (2001), no. 1, pp. 1–36.

[3] Alexey Mkrtychev, *Models for the Logic of Proofs*, ***LFCS***, Lecture Notes in Computer Science, vol. 1234 (1997), pp. 266–275.

► DANIEL LEIVANT, *Second order logic and the metamathematics of logics of programs*.
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Deductive reasoning about imperative programs has been formalized in various settings, using e.g., programs as modalities, temporal modalities, fixpoint operators, first order renditions of program-semantics, and infinitary logic. Pursuing ideas first presented in [1], we use instead second order logic (with restricted Comprehension), and establish a gamut of new as well as known meta-mathematical properties of modal logics of programs. In particular, we characterize the proof theoretic power of first order Dynamic Logics, and of their extension with a fixpoint operator.

[1] Daniel Leivant, *Logical and mathematical reasoning about programs*, ***Conference Record of the Twelfth Annual Symposium on Principles of Programming Languages***, ACM, New York, 1985, pp. 132–140.

► BENEDIKT LÖWE, *Measure assignments and Kleinberg sequences*.
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Jackson has developed his description theory in order to compute the projective ordinals in the theory ZF + AD. The main technical tool of description theory is the assignment of measures (more exactly, of descriptions) to cardinals below the supremum of the projective ordinals.

In this talk, we shall discuss further consequences of the existence of such a measure assignment. As a corollary, we develop algorithms to compute the regular cardinals, all cofinalities and all Kleinberg sequences below the supremum of the projective ordinals.

The work described is joint work with Steve Jackson (Denton, TX).

► YEVGENIY MAKAROV, *Classical proofs viewed as functional programs with control operators*.
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In [2], Griffin extended Curry-Howard isomorphism to classical logic by observing that inference by contradiction corresponds to Felleisen's control operator $\mathcal{C}$. However, the exact correspondence between the program extracted from a classical derivation and the constructive content of its derived formula has remained unclear to date.

We formulate and prove such a correspondence for proofs of Π^0_2 formulas. The main idea is the following. Fix a closed formula F not containing $\bot$ and replace all occurrences of $\bot$ in a classical natural deduction derivation $\mathcal{D}$ by F. This, of course, makes $\mathcal{D}$ not a valid derivation because now it may contain nodes of the form

$$\frac{F}{D} \quad \text{and} \quad \frac{(D \to F) \to F}{D},$$

which correspond to intuitionistic and classical rules for negation, respectively. However, if $\mathcal{D}$ is a closed derivation of F, these nodes may be eliminated by conversions on derivations which by Curry-Howard isomorphism correspond to conversions for control operators $\mathcal{A}$ and $\mathcal{C}$.

We propose some extensions to Griffin's method that handle equality and atomic inference rules. Applying this method, we are able to extract SCHEME programs from a broad class of classical proofs including several interesting ones considered in [1].

[1] Ulrich Berger, Wilfried Buchholz, and Helmut Schwichtenberg, *Refined program extraction from classical proofs*, ***Annals of Pure and Applied Logic***, vol. 114 (2002), no. 1–3, pp. 3–25.

[2] Timothy G. Griffin, *Formulas-as-types notion of control*, ***Conference record of the seventeenth annual ACM symposium on principles of programming languages***, 1990, pp. 47–58.

► RUSSELL MILLER, *The curious case of order-computable sets*.
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Let S be a subset of ω, and consider the structure $(\omega, <, S)$, in the language of linear orders with an additional unary predicate. We say that S is *order-computable* if this structure is computably presentable, i.e., if there is a computable set C and a computable order $\prec$ on ω such that $(\omega, \prec, C) \cong (\omega, <, S)$.

This simple concept resists any straightforward characterization by purely computability-theoretic properties. We present a survey of results about order-computable sets, with proofs described or sketched as time permits, including the following. All low c.e. sets are order-computable, but there exist c.e. sets and low d.c.e. sets which are not. Every n-c.e. set is Turing-equivalent to an n-c.e. order-computable set, and similarly for ω-c.e. sets. However, there exist Turing degrees below $\mathbf{0}'$ containing no order-computable set. There also exist (noncomputable) Turing degrees containing only order-computable sets. No 1-random set is order-computable. Finally, we prove that there exist an order-computable set and an order-noncomputable set which are computably isomorphic to each other. This last result suggests the extent to which the property of order-computability differs from most computability-theoretic properties.

This work is joint with Denis Hirschfeldt and Sergey Podzorov.

► PAVEL NAUMOV, *On modal logics of partial computable functions*.
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The classical propositional logic is sound and complete with respect to the set semantics τ under which propositional connectives conjunction, disjunction, and negations are interpreted as operations intersection, union, and complement on subsets of any infinite universe. In particular, the universe could be the set of all words in some alphabet Σ. We extend the language of the classical propositional logic by a new binary modality $\rhd$. The set semantics is extended to interpret modality $\rhd$ as the following operation on subsets of Σ^*: $\tau(\phi \rhd \psi)$ is equal to the set of all $x \in \Sigma^*$ such that for any $y \in \tau(\phi)$ if Turing machine x terminates on input y then it returns an element of $\tau(\psi)$ as output. The modal logic of partial computable functions is the set of all propositional modal formulas whose interpretation is equal to Σ^* for any interpretation of propositional variables.

Theorem 1. *The modal logic of partial computable functions is an extension of the classical propositional logic by the following axioms*:

$$\phi \rhd \psi \to (\phi \rhd \chi \to \phi \rhd (\psi \wedge \chi)) \quad \text{(C)}$$

$$\phi \rhd \psi \to (\chi \rhd \psi \to (\phi \vee \chi) \rhd \psi) \quad \text{(D)}$$

$$\bot \rhd \phi \quad \text{(F)} \qquad\qquad \phi \rhd \top \quad \text{(T)}$$

and, in addition to Modus Ponens, the following inference rules:

$$\frac{\phi \to \psi}{\psi \rhd \chi \to \phi \rhd \chi} \quad \text{(LM)} \qquad\qquad \frac{\phi \to \psi}{\chi \rhd \phi \to \chi \rhd \psi} \quad \text{(RM)}$$

THEOREM 2. *The modal logic of partial computable functions is decidable.*

▶ MICHAEL RAY OLIVER, *Many nonisomorphic Boolean algebras $\mathcal{P}(\omega)/\mathcal{I}$.*
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This talk presents a summary of work to appear in the JSL; time permitting; it may also touch on recent joint work with Su Gao.

I examine the question of how many Boolean algebras, distinct up to isomorphism, that are quotients of the powerset of the naturals by Borel ideals, can be proved to exist in ZFC alone. The maximum possible value is easily seen to be the cardinality of the continuum $2^{\aleph_0}$; earlier work by Ilijas Farah had shown that this was the value in models of Martin's Maximum or some similar forcing axiom, but it was open whether there could be fewer in models of the Continuum Hypothesis.

I develop and apply a new technique for constructing many ideals whose quotients must be nonisomorphic in any model of ZFC. The technique depends on isolating a kind of ideal, called shallow, that can be distinguished from the ideal of all finite sets even after any isomorphic embedding, and then piecing together various copies of the ideal of all finite sets using distinct shallow ideals. In this way we are able to demonstrate that there are continuum-many distinct quotients by Borel ideals, indeed by analytic P-ideals, and in fact that there is in an appropriate sense a Borel embedding of the Vitali equivalence relation into the equivalence relation of isomorphism of quotients by analytic P-ideals.

Recent work with Gao examines whether an arbitrary Borel equivalence relation may be embedded into the isomorphism relation on quotients by Borel ideals.

▶ ERIC PACUIT AND ROHIT PARIKH, *A logic for communication graphs (Preliminary report).*
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The *Topologic* of Moss and Parikh is extended to the case of many agents $\mathcal{A} = (1, \dots, n)$ who are assumed to have some private information at the outset, but may refine their information by acquiring information possessed by other agents, possibly via other agents. $\mathcal{P}_i$ is i's information partition; $\hat{\mathcal{P}} = (\mathcal{P}_1, \dots, \mathcal{P}_n)$. Partitions refine when information is exchanged.

In the *communication graph* on $\mathcal{A}$, an edge (i, j) means that agent i can directly receive information from agent j. W is the set of possible worlds and V some valuation on W of the propositional symbols, understood by all agents, but with only specific agents knowing their actual values at worlds $x \in W$.

L_0 is the propositional (base) language, L' its closure under the operators K_i, and L *its* closure (only) under $\diamond$ and the boolean operators. Given a partition $\mathcal{P}$, $\mathcal{P}(x)$ is that cell of the partition $\mathcal{P}$ in which x lies.

$x, \hat{\mathcal{P}} \models P$ means that $V(x, P) = 1$, booleans are interpreted n the obvious way.
$x, \hat{\mathcal{P}} \models K_i\phi$ iff $(\forall y \in \mathcal{P}_i(x), y, \hat{\mathcal{P}} \models \phi)$
$x, \hat{\mathcal{P}} \models \diamond\phi$ iff ($\exists\hat{\mathcal{Q}}$ such that $\hat{\mathcal{Q}}$ refined $\hat{\mathcal{P}}$ and $x, \hat{\mathcal{Q}} \models \phi$)

The formula scheme $K_i(\phi) \to \diamond K_j(\phi)$ holds iff agent j can directly or indirectly acquire information possessed by i. The validities of *Topologic* remain valid and the communication graph is completely determined by the validities of the resulting logic. Applications of our logic to the Bush–Tenet dilemma are obvious.

[1] MOSS AND PARIKH, *Topological reasoning and the logic of knowledge*, ***TARK IV***,

Y. Moses (editor), Morgan Kaufmann, 1992.

▶ ALEXANDER RAICHEV, *Relative randomness and real closed fields*.
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We prove that for all $\beta \in {}^{\omega}2$, $\mathcal{R}^{\beta} := \langle \mathbb{R}^{\beta}, +, \cdot, < \rangle$ is a countable real closed field, where $\mathbb{R}^{\beta}$ is the set of all reals less random than β in the sense of rK-reducibility. This generalizes the fact that the computable reals form a countable real closed field. One consequence of this and its proof is that the d.c.e. reals form a real closed subfield of the field of reals less random than Ω, Chaitin's random real.

[1] K. Ambos-Spies, K. Weihrauch, and X. Zheng *Weakly computable real numbers*, ***Journal of Complexity***, vol. 16 (2000), pp. 676–690.

[2] R. G. Downey, D. Hirschfeldt, and G. Laforte, *Randomness and reducibility*, ***Journal of Computer and System Sciences***, vol. 68 (2004), pp. 96–114.

[3] D. Marker, *Model theory: An introduction*, Springer-Verlag, Berlin/Heidelberg, Germany and New York, USA, 2002.

[4] M. B. Pour-El and J. I. Richards, *Computability in analysis and physics*, Springer-Verlag, Berlin/Heidelberg, Germany and New York, USA, 1989.

▶ MARTIN K. SOLOMON, *Some remarks on Gödelian philosophy*.
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It is considered whether Kurt Gödel's philosophy of mathematics, especially as expressed in [2, p. 484], taken together with footnote 11 in [2, p. 475], can be viewed as a form of "epistemological structuralism" similar to (but more optimistic than) what is given in [5, p. 134]. Compare also with Gödel's letter to Greenberg in [1, p. 454].

It is shown that both Gödel's general relativity results concerning rotating universes, and certain special relativity results concerning tachyon inertial frames [4] can be used to support either the reality of time or the Gödelian view on the ideal nature of time, depending on one's metaphysical orientation concerning the relationship between the possible and the actual.

Similarly, it follows from [3] that the limiting theorems of logic can either be used to argue for a materialistic view of mind, or to argue (as Gödel does) for the nonmaterialistic view that the mind transcends the physical brain.

Finally, it is considered whether the sum of Gödel's published philosophy provides a sort of optimistic neo-Kantian epistemology superimposed on a Platonic metaphysics.

[1] S. Feferman et al., editors, ***K. Gödel, Collected works IV: correspondence A–G***, Oxford University Press, Oxford, 2003.

[2] K. Gödel, *What is Cantor's continuum hypothesis?*, P. Benacerraf and H. Putnam (editors), ***Philosophy of mathematics: selected readings***, 2nd edition, Cambridge University Press, Cambridge, UK, 1983.

[3] A. E. Lyngzeidetson and M. K. Solomon, *Abstract complexity theory and the mind-machine problem*, ***The British Journal for the Philosophy of Science***, vol. 45 (1994), pp. 549–554.

[4] L. Parker, *Faster-than-light inertial frames and tachyons*, ***Physical Review***, vol. 188 (1969), pp. 2287–2292.

[5] M. Steiner, ***Mathematical knowledge***, Cornell University Press, Ithaca, NY, 1975.

▶ ELISA VASQUEZ, *An application of Crofton's formula to o-minimal structures*.
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Any definable, bounded set A of an o-minimal expansion of $(\mathbb{R}, <, 0, +, -, \cdot, 1)$ can be decomposed into finitely many definable sets A_i such that there is a constant K and a definable families of curves λ_i in each A_i with the property that any pair of points $x, y \in A_i$ can be joined by a curve γ in the family λ_i with $l(\gamma) \leq K|x - y|$.

▶ MICHAEL A. WARREN, *Predicative categories of classes*.
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Joyal and Moerdijk [3] initiated the category theoretic study of set theory and provided a category theoretic model of **IZF**. Recently this research has been extended by Awodey *et al.* [2], who have demonstrated the importance of *categories of classes* for providing models of set theories. Roughly, a category of classes is a Heyting category endowed with a system of distinguished *small maps*, a system of *small powerobjects*, and a *universal object* U. Such categories arise naturally as models of a particular intuitionistic set theory (**bIST**). Interestingly, any elementary topos $\mathcal{E}$ may be completed to a category of classes $\mathbf{Idl}(\mathcal{E})$, the category of ideals of $\mathcal{E}$, and **bIST** is complete with respect to such models: for any formula φ, if $\mathbf{Idl}(\mathcal{E})$ satisfies φ, for every elementary topos $\mathcal{E}$, then φ is provable in **bIST**.

However, topoi (and **bIST**) are impredicative and so the question naturally arises whether similar results to the aforementioned are possible for predicative set theories such as **CZF** (cf. [1]). I show that there is an affirmative answer to this question. In particular, any locally cartesian closed Heyting pretopos $\mathcal{C}$ may be completed to a 'predicative category of classes' $\mathbf{Idl}(\mathcal{C})$. Regarding such categories $\mathcal{C}$ as 'predicative topoi' I arrive at an analogous completeness result for a predicative set theory **bCST**.

[1] PETER ACZEL AND MICHAEL RATHJEN, *Notes on constructive set theory*, Technical Report 40, Institut Mittag–Leffler (Royal Swedish Academy of Sciences), 2001, http://www.ml.kva.se/preprints/archive2000-2001.php.

[2] STEVE AWODEY, CARSTEN BUTZ, ALEX SIMPSON, AND THOMAS STREICHER, *Relating set theory and topos theory using categories of classes*, Technical Report CMU-PHIL-116, Carnegie Mellon University, June 2003, www.andrew.cmu.edu/~awodey/.

[3] A. JOYAL AND I. MOERDIJK, ***Algebraic set theory***, Cambridge University Press, Cambridge, 1995.

▶ REBECCA WEBER, *Invariance and orbits in the lattice of* Π^0_1 *classes*.
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The lattice of Π^0_1 classes, $\mathcal{E}_\Pi$, has been well-studied, but little is known about the orbits of its elements and degree invariant classes. This is not the case with $\mathcal{E}$, the lattice of c.e. sets. We present a way to transfer orbits from $\mathcal{E}$ to invariant classes of $\mathcal{E}_\Pi$ via a definable quotient substructure of $\mathcal{E}_\Pi$ which is isomorphic to $\mathcal{E}^*$; that is, $\mathcal{E}$ modulo finite difference. Unfortunately the method of obtaining invariant classes does not also result in orbits, except possibly in cases where all elements of the orbit are of maximal Cantor-Bendixson rank. We will discuss our continuing work in this area.