Abstracts of invited talks

► JOHN BALDWIN, *Categoricity transfer in infinitary logic*.
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We distinguish two themes in generalizing Morley's theorem to infinitary logic. One is focused primarily on the logic $L_{\omega_1,\omega}$ and does not assume that there are arbitrarily large models. Major steps in the analysis begin with the hypothesis that a sentence has few models in small cardinalities (below $\aleph_\omega$). This leads to the theory of excellence, which has applications in understanding complex exponentiation. The other theme concerns the more general setting of Abstract Elementary Classes (AEC): a class with a notion of strong submodel that satisfies a variant of the Jónsson axioms. Most progress has been made by assuming both the amalgamation property and that the class has arbitrarily large models, so the Ehrenfeucht-Mostowski technology can be invoked. We will expound recent work by Shelah, Grossberg and VanDieren towards the conjecture: if an AEC is categorical in a sufficiently large cardinal, it is categorical in all sufficiently large cardinals.

► LEV BEKLEMISHEV, *From provability logic to combinatorial independence results*.
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The questions of naturality and canonicity of proofs lie at the heart of difficult conceptual problems in proof theory. One possible way to approach these questions is to consider them from a more abstract, algebraic standpoint.

Provability algebras are boolean algebras with additional "provability" operators developed to provide an abstract approach to proof-theoretic analysis. The subject brings together traditional methods of proof theory and those of modal provability logic. Potential aims are, for example,

- getting insight from a new perspective into the foundational questions such as the problem of canonicity of ordinal notations.
- providing a technically simple and clean treatment of traditional proof-theoretic results such as consistency proofs, combinatorial independent principles, etc.

We give an overview of these methods and formulate some simple statements of combinatorial character independent from Peano arithmetic arising from the provability algebra approach. Interesting connections with some examples of Friedman and Schütte–Simpson and Buchholz will be indicated.

► MICHAEL HALLETT, *Reflections on the purity of method in Hilbert's* Grundlagen der Geometrie.
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Hilbert's *Grundlagen der Geometrie* presents a new way of looking at the foundations of mathematical theories, one which has become the dominant one in foundational investigations. This approach seems incompatible with traditional 'purity of method' concerns which concentrate on the supposed superiority of certain theoretical frameworks. Nevertheless, this paper argues that this impression is only partially correct. (1) It illustrates several cases where Hilbert uses 'purity' questions, traditionally posed, to provide new foundational information. (2) But more strongly, the investigations throw open the question of the mathematical presuppositions of foundational investigation itself.

▶ VALENTINA S. HARIZANOV, *Effectiveness in algebraic structures*.
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We apply ideas, notions and methods from computability theory to study algorithmic phenomena (effectiveness) in countable algebraic structures. We focus on the computability theoretic complexity of structures, the complexity of their isomorphisms, and of relations that are not named in the languages of the structures. Since complexity can be described syntactically (for example, using computable infinitary formulae) and measured semantically (for example, using Turing degrees), this study also relates definability to computability. Classically isomorphic structures can have very different algorithmic properties. When complexity bounds for some property of a structure are preserved under isomorphisms, we call the bounds intrinsic. We are interested in intrinsically and nonintrinsically complicated relations on computable and hyperarithmetical structures. Results establishing equivalences of syntactic conditions and corresponding algorithmic properties in computable isomorphic copies of a computable structure $\mathcal{A}$ usually require additional effectiveness in $\mathcal{A}$. If we allow arbitrary isomorphic copies of $\mathcal{A}$, we obtain analogous results for relativized algorithmic properties, without requiring additional effectiveness. We investigate both general structures and specific ones, such as partial and linear orderings, Boolean algebras, and groups.

[1] C. J. Ash and J. F. Knight, ***Computable structures and the hyperarithmetical hierarchy***, Elsevier, 2000.

[2] S. S. Goncharov, V. S. Harizanov, J. F. Knight, and C. F. D. McCoy, *Simple and immune relations on countable structures*. ***Archive for Mathematical Logic***, vol. 42 (2003), pp. 279–291.

[3] S. Goncharov, V. Harizanov, J. Knight, C. McCoy, R. Miller, and R. Solomon, *Enumerations in computable structure theory*, ***Annals of Pure and Applied Logic***, submitted.

[4] S. S. Goncharov, V. S. Harizanov, J. F. Knight, and R. A. Shore, Π^1_1 *relations and paths through* $\mathcal{O}$, ***The Journal of Symbolic Logic***, to appear.

[5] V. Harizanov and R. Miller, *Spectra of structures and relations*, preprint.

▶ STEVE JACKSON, *Regular markers and countable equivalence relations*.
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We use the existence of markers with regular geometric properties to study countable Borel equivalence relations. In particular we obtain a new proof of the hyperfiniteness of Z^n actions and the existence of a continuous embedding of 2^Z into E_0.

▶ KENNETH KUNEN, *Elementary submodels in topology and analysis*.
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In this technique, one takes an elementary submodel of the universe (or, to be formal, of some suitably large $H(\theta)$), to facilitate some combinatorial proof. The basic idea goes back to Gödel's proof of the GCH in L, and is now well-known in set-theoretic arguments. The talk will discuss some applications of this technique to general topology and functional analysis.

▶ PENELOPE MADDY, *Mathematical existence*.
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This talk will explore some interrelated issues of mathematical method, truth and existence.

► ANGUS MACINTYRE, *Schanuel's Conjecture and its significance for the logic of the real and complex exponentials*.
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I will survey the use of Schanuel's Conjecture for the metamathematics of the exponential function. The first applications were to decidability, culminating in its use in the decidability of the real exponential function. Here the key point is that the conjecture links naturally to a normal form for existential formulas, and a notion of exponential-algebraic. The more recent work of Zilber has a different flavour, as the basic facts about the two exponential's are so different. I will discuss the relation between Zilber's exponential and the classical complex exponential.

► JOSEPH S. MILLER, *Measuring the randomness of random reals*.
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What does it mean to say that one real number is *more random* than another? A variety of quasi-orders have been introduced to answer this question; in other words, to measure the "degree of randomness" of a real. We argue that the reducibilities introduced by Solovay (unpublished) and by Downey, Hirschfeldt, and LaForte [1] are too strong to compare the randomness of random reals. We examine two weaker measures of randomness (already introduced by André Nies in the study of highly non-random sets) and relate the new and old reducibilities. This is done by proving that certain properties of a real can be inferred from the (plain or prefix-free) Kolmogorov complexity of its initial segments.

Closely related to this investigation is the inverse correlation between the degree of randomness of a real and its power as an oracle. Although a random real has "high information content", it need not have high Turing complexity. For sufficiently random reals, the opposite is true. We support this thesis with several results, both old and new. For example, a 1-random real Turing below an n-random is also n-random; so randomness strength can be gained by bounding computational strength from above.

This talk includes joint work with Liang Yu.

[1] ROD G. DOWNEY, DENIS R. HIRSCHFELDT, AND GEOFF LAFORTE, *Randomness and reducibility*, ***Mathematical foundations of computer science 2001*** (***Mariánské Lázně***), Lecture Notes in Computer Science, vol. 2136, Springer Berlin, 2001, pp. 316–327.

► MICHAEL RATHJEN, *Equiconsistency results and open problems in constructive and intuitionistic set theories*.
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Constructive Zermelo-Fraenkel Set Theory, CZF, has emerged as a standard reference theory that relates to constructive predicative mathematics as ZFC relates to classical Cantorian mathematics. A hallmark of this theory is that it possesses a type-theoretic model.

Following a brief review of the history of CZF, the talk will be concerned with equiconsistency results and open problems relating to CZF, IZF and even ZF; principally questions concerning choice principles (presentation axiom, axiom of multiple choice), large set axioms (the regular extension axiom and its variants), subset collection versus exponentiation, and replacement versus collection.

► PAUL VITANYI, *Extracting meaning with Kolmogorov and Shannon*.
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As perhaps the last mathematical innovation of an extraordinary scientific career, Kolmogorov in 1974 proposed to found statistical theory on finite combinatorial and computability principles independent of probabilistic assumptions, as the relation between the individual data and its explanation (model), expressed by Kolmogorov's structure function. It turns out that this proposal is formally a Shannon's rate distortion function for individual data and related to lossy compression.

In classical probabilistic statistics the goodness of the selection process is measured in terms of expectations over probabilistic ensembles. For current applications, average relations are often irrelevant, since the part of the support of the probability density function that will ever be observed has about zero measure. This may be the case in, for example, complex video and sound analysis. There arises the problem that for individual cases the selection performance may be bad although the performance is good on average, or vice versa. There is also the problem of what probability means, whether it is subjective, objective, or exists at all. Kolmogorov's proposal outlined strives for the firmer and less contentious ground expressed in finite combinatorics and effective computation.

This Kolmogorov's structure function, its variations and its relation to model selection, have obtained some notoriety (many papers and Cover and Thomas textbook on Information Theory) but have not before been comprehensively analyzed and understood. It has always been questioned why Kolmogorov chose to focus on the a mysterious function denoted as h_x, rather than on a more evident function denoted as β_x (for details see paper referred to below). Our main result, with the beauty of truth, justifies Kolmogorov's intuition. One easily stated consequence is: For all data, minimizing a two-part code consisting of one part model description and one part data-to-model code (essentially the celebrated MDL code), subject to a given model-complexity constraint, as well as minimizing the one-part code consisting of just the data-to-model code (essentially the maximum likelihood estimator), *in every case* (and not only with high probability) selects a model that is a "best explanation" of the data within the given constraint. In particular, when the "true" model that generated the data is not in the model class considered, then the ML or MDL estimator still give a model that "best fits" the data. This notion of "best explanation" and "best fit" is understood in the sense that the data is "most typical" for the selected model in a rigorous mathematical sense that is discussed below. A practical consequence is as follows: While the best fit (minimal randomness deficiency under complexity constraints on the model) cannot be computationally monotonically approximated, we can monotonically minimize the two-part code, or the one-part code, and thus monotonically approximate *implicitly* the best fitting model. But this should be sufficient: we want the best model rather than a number that measures its goodness. These results open the possibility of model selection and prediction that are best possible for *individual* data samples, and thus usher in a completely new era of statistical inference that is *always* best rather than *expected*. It turns out that the structure function can be viewed as a rate distortion function for individual data. With this change of viewpoint we connect Kolmogorov's approach to non-probabilistic statistics and model selection with Claude Shannon's rate distortion theory and the theory of lossy compression. This leads to a theory of rate distortion for individual data and a general class of structure functions. Based on joint work with Nikolai Vereshchagin presented partially at the 47th IEEE Symposium on Foundations of Computer Science, 2002, Vancouver, Canada. See paper at `http://www.cwi.nl/~paulv/selection.html` or `http://arXiv.org/abs/cs.CC/0204037` and recent work with Peter Grunwald and Nikolai Vereshchagin.