Abstracts of invited special lectures

► WILLIAM A. HOWARD, *Aspects of proof theory and foundations, 1950–1985*.
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The motivating question for me in the early 1950s was as follows. Functional analysis, developed within the framework of set theory including the axiom of choice, is very abstract. What is the relation between this and the physical world? Even Peano arithmetic, PA, is problematic, since it is based on an idea of truth whose empirical meaning is not clear. Two tools for the investigation of questions of this kind within the framework of constructive reasoning have been: the descending chain principle within some system of constructive ordinal notations (Gentzen, 1938), and Gödel's interpretation of intuitionistic systems by means of functionals of finite type (1958).

New perspectives were opened up by Spector (1962) and Kreisel. This led to the pursuit of the following program. Take a suitable subsystem of classical analysis, translate this into an intuitionistic system, then give a proof theoretic treatment of the latter by means of Gödel's functional interpretation, using the appropriate system of functionals of finite type.

When one considers functionals of finite type within a constructive framework, the question arises: What is a type? In investigating this question, Curry found a striking relation between the types of elementary functionals and the theorems of positive implicational logic. This generalizes to the intuitionistic predicate calculus and to various systems of intuitionistic mathematics. The relation between types and intuitionistic logic has been found useful in the study of strongly typed programming languages.

In summary, the philosophical goals have been only partially achieved, and may very well be unachievable, but there have been some useful practical consequences.

► JOHN MCCARTHY, *What more does artificial intelligence want from mathematical logic?*
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Leibniz, Boole and Frege all expected the mathematical logic they were developing to be applicable to human affairs outside mathematics and science. Leibniz explicitly wanted to replace disputation by calculation. Unfortunately, this didn't happen.

A complete system of first order logic was essentially developed by Frege and proved complete by Gödel. The Zermelo-Fraenkel set theory formally adequate for classical mathematics was available by 1920. However, no-one was able to replace disputation by calculation by formulating common sense facts and arguments in mathematical logic.

Logical artificial intelligence (AI) since 1959 has represented facts about the common sense world in logical languages and has expressed rules giving the effects of actions and other events as sentences in languages of mostly first order logic. The plan was and is for a computer program to decide what to do by logically inferring that a certain reasoning strategy was appropriate for attempting to achieve a certain goal.

Difficulties arose that required extensions of logical formalisms. The most important extension is to nonmonotonic reasoning, in which allows brave inferences that may later have to be withdrawn. There are several systems for this, all of which can be understood in terms of preferred models. This lecture will mention circumscription, essentially minimization of a tuple of predicates subject to an axiom constraining the interpretations, with some symbols variable and others held constant.

More than just nonmonotonic reasoning is needed, but then matters become controversial even within AI. The lecture will discuss individual concepts and propositions as first order

objects, contexts as objects, and approximate objects without defined extensions.

All these matters come up in connection with the common sense informatic situation that people face in trying to achieve goals. It is not decided in advance what concepts and facts are relevant.

Human-level logical AI needs the following, and logicians have helped and can help more.

1. Languages covering more and more of common sense knowledge. This work has gone slowly, because the needed concepts are often incompletely definable, i.e., don't have if-and-only-if definitions.

2. A "heavy duty" set theory within which a system can do its reasoning. Present systems are limited in scope and require too much human intervention. However, some substantial theorems, e.g., Gödel's first incompleteness theorem, have been proved by interactive systems where a person provides guidance.

3. Logical problem solving programs that can use domain dependent heuristic information. This is necessary, because present general purpose problem solvers generate too much junk.

4. A language capable of expressing meta-reasoning about theories in the language. Almost all research in mathematical logic has involved informal reasoning about theories from above rather than formal reasoning within the theory. A human-level system must not require human supervision. Making such a system requires accepting incompleteness and avoiding paradoxes by suitable weakening of axioms.

▶ DANA S. SCOTT, *Sets, topologies, categories*.
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Sets and arbitrary maps form a rich category, as everyone knows, but it may not be so clear that there are natural subcategories (i.e., with fewer maps). For example, every set has an intrinsic, generally non-discrete topology. The reason is that every powerset has a natural topology (in fact, more than one, but we concentrate on the non-Hausdorff topology of "finite information"), and, in the usual formulation of Zermelo-Fraenkel set theory (ZF), every set is a subset of the powerset of its unionset. In this way a set inherits a subspace topology, and consequently set inclusion becomes the same as subspace inclusion. The intrinsic topology is easily defined, as is the notion of continuous function (a property of the usual set-theoretic notion of function), and the category of sets and continuous functions is equivalent to the category of topological T_0-spaces and continuous functions as usually defined in pointset topology. This topological category of sets can be expanded to a category of equivalence relations and continuous, equivariant functions by equally easy set-theoretic definitions, and the resulting category has interesting closure properties and applications not seen in the topological category. In intuitionistic Zermelo-Fraenkel (IZF) it is even possible to postulate the existence of a small subcategory with extensive closure properties, an inconsistent assumption in classical logic. The lecture is intended as a survey.