Convex Hull for Dynamic Data

Convex Hull and Parallel Tree Contraction

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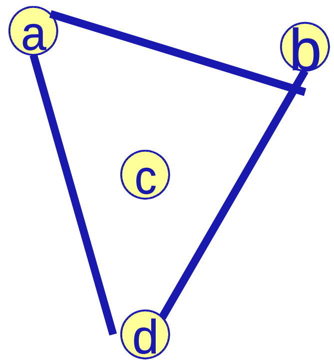
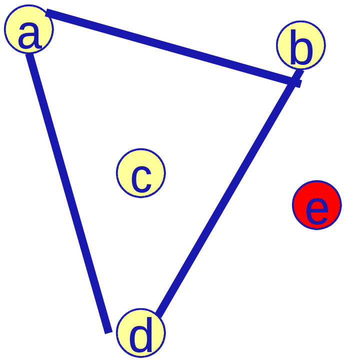
(joint work with Umut Acar, and Guy Blelloch)

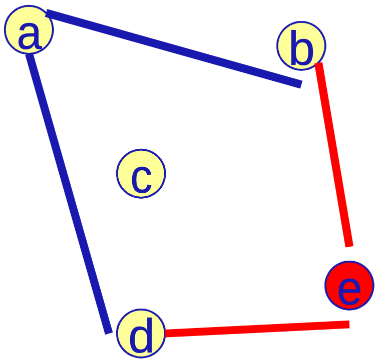
Motivation

- Application data is dynamic
  - word processors: slowly changing text
  - graphics: render similar images
  - mobile phone networks: continuously moving hosts
- Important to handle dynamic data efficiently

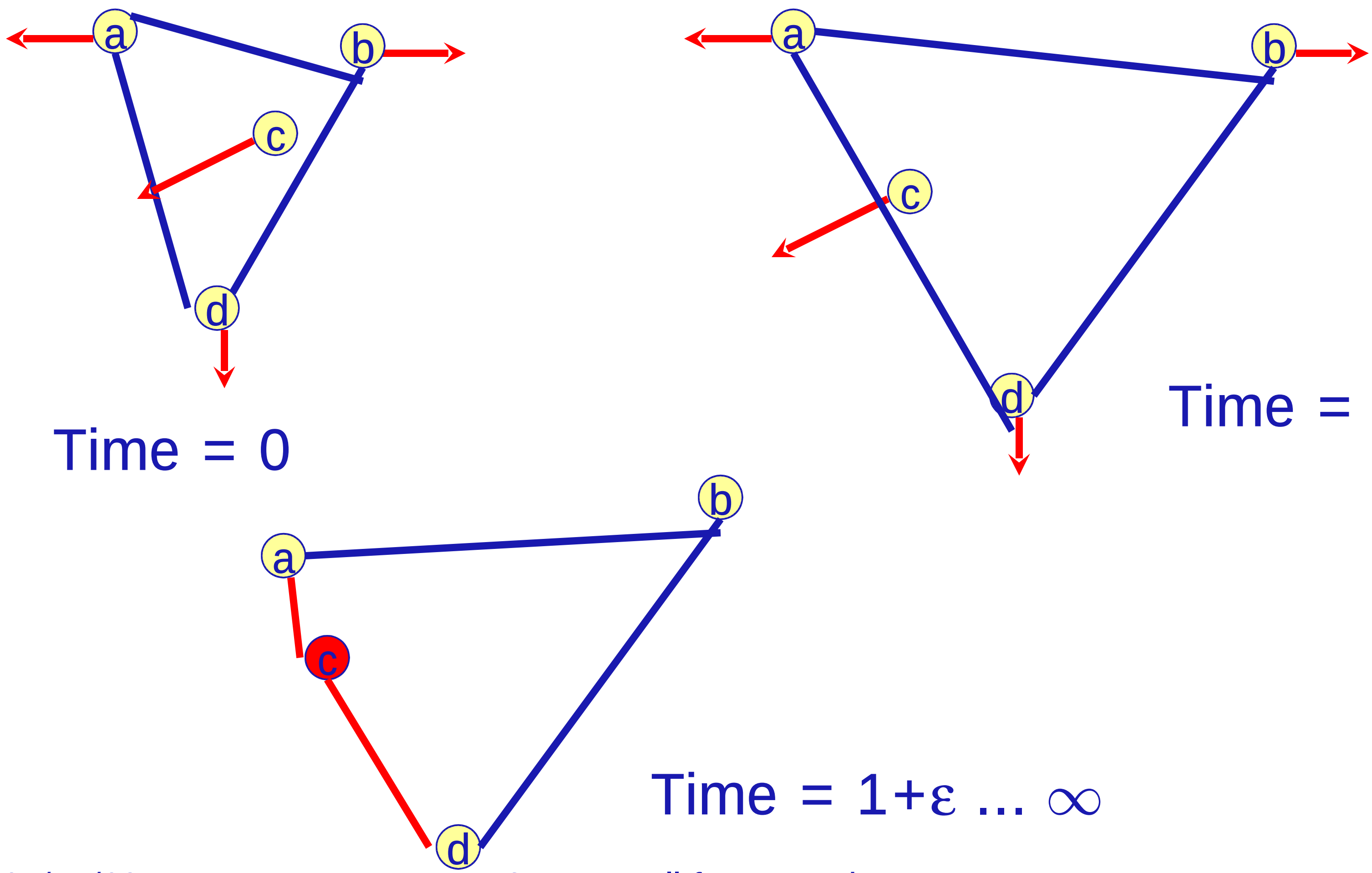
Dynamic Algorithms: Changing Data







Kinetic Algorithms: Moving Data



How to invent Dynamic/Kinetic Algorithms

- Just like any other algorithm. Think, ponder, divide, conquer…
- Or, use adaptivity…

Adaptivity

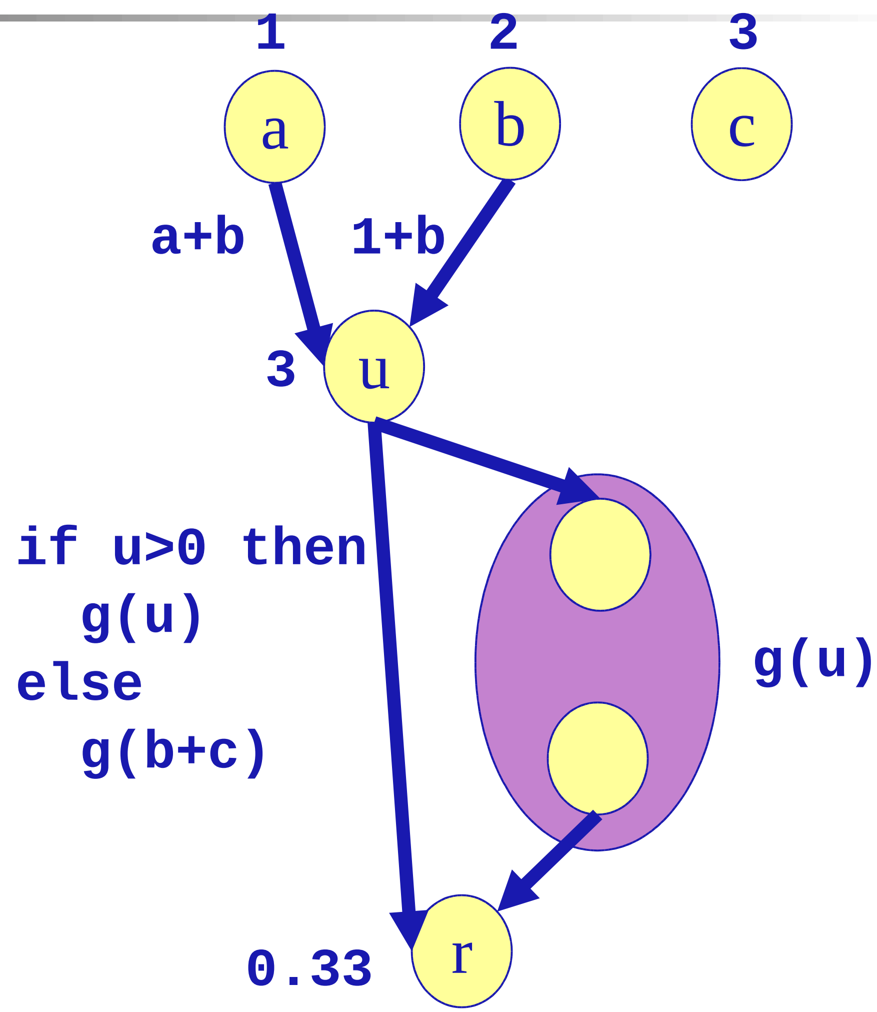
- Makes a standard algorithm dynamic or kinetic
- Requires little change to the standard algorithm
- Can be done semi-automatically
- Not all algorithms yield efficient adaptive algorithms
  - Will talk about this more

How does Adaptivity Work?

- Represent a computation with a dynamic dependency graph
- nodes ℑ data, edges = dependencies
- Sources = input, sinks = output,
- The user can
  - change the input,
  - update the output
- Update:
  - Take a changed node,
  - Update all its children (the children are now changed)
  - Repeat until no more changed nodes

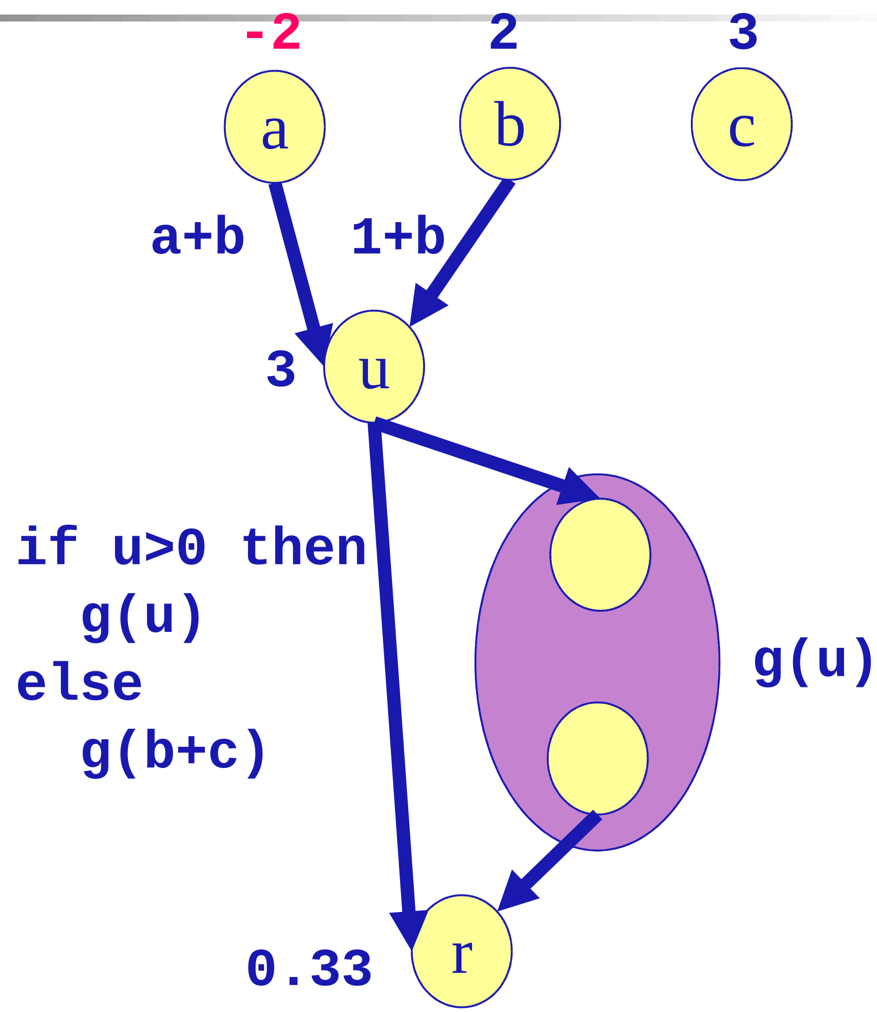
Adaptivity Example

```
fun f (a,b,c) =
let
  u = a+b
in
  if (u > 0) then
    g(u)
  else
    g(b+c)
end
```



Adaptivity Example: Change

```
fun f (a,b,c) =
let
  u = a+b
in
  if (u > 0) then
    g(u)
  else
    g(b+c)
end
```



Update

```
fun f (a,b,c) =
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end
```

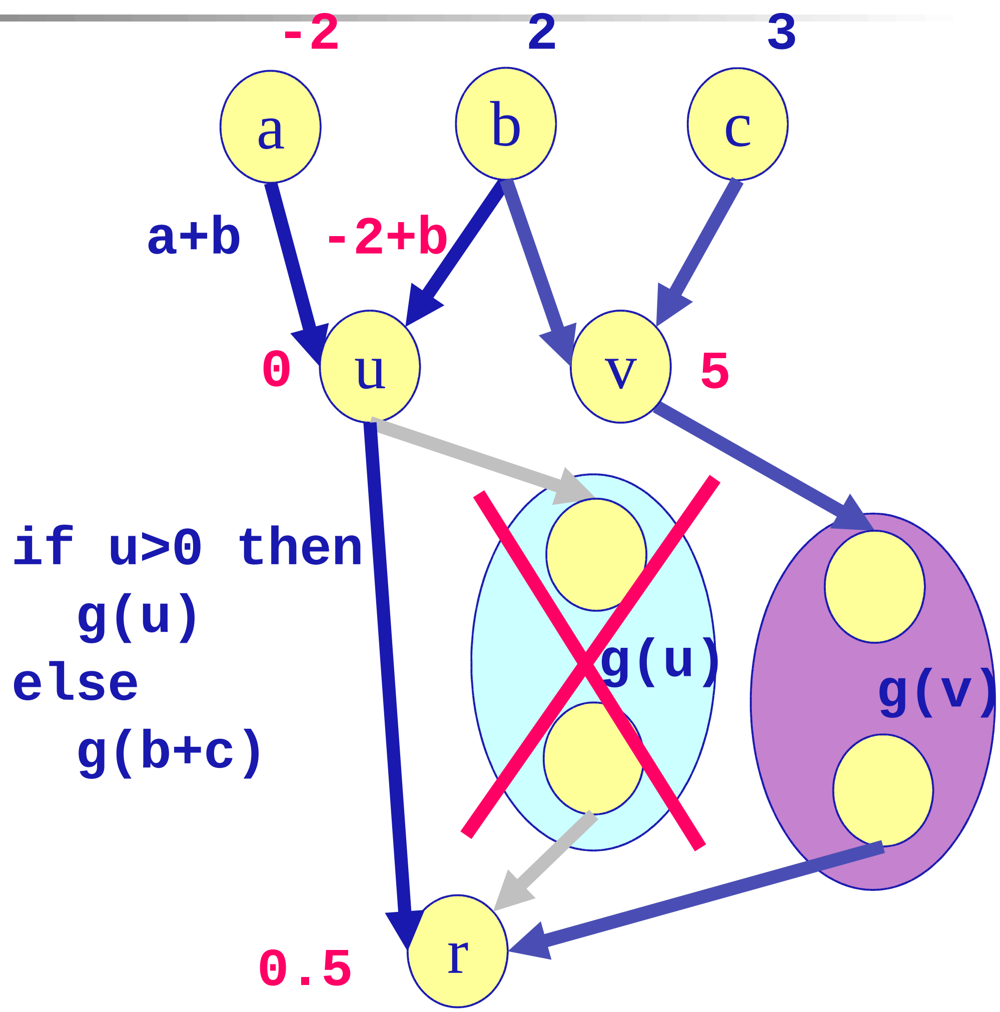

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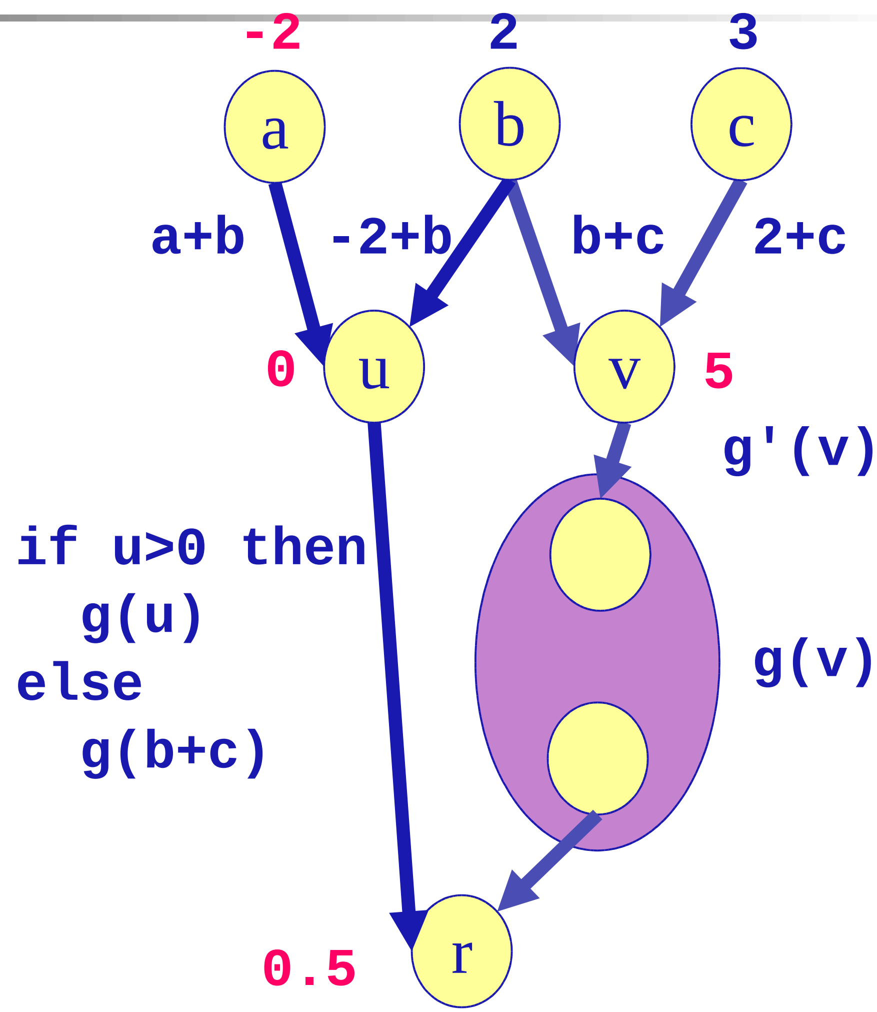


Adaptivity and Stability

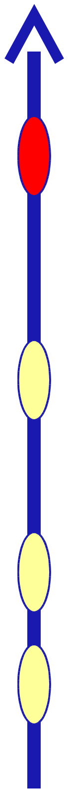
- Adaptivity updates the result by rerunning the parts of the computation affected by the input change
- It is efficient when the computation is “stable”,I.e., computations on “similar” inputs are “similar”

- We apply the adaptivity technique to convex hulls
- Result: Efficient Dynamic and Kinetic convex hulls

1-D Convex Hull: Max and Min

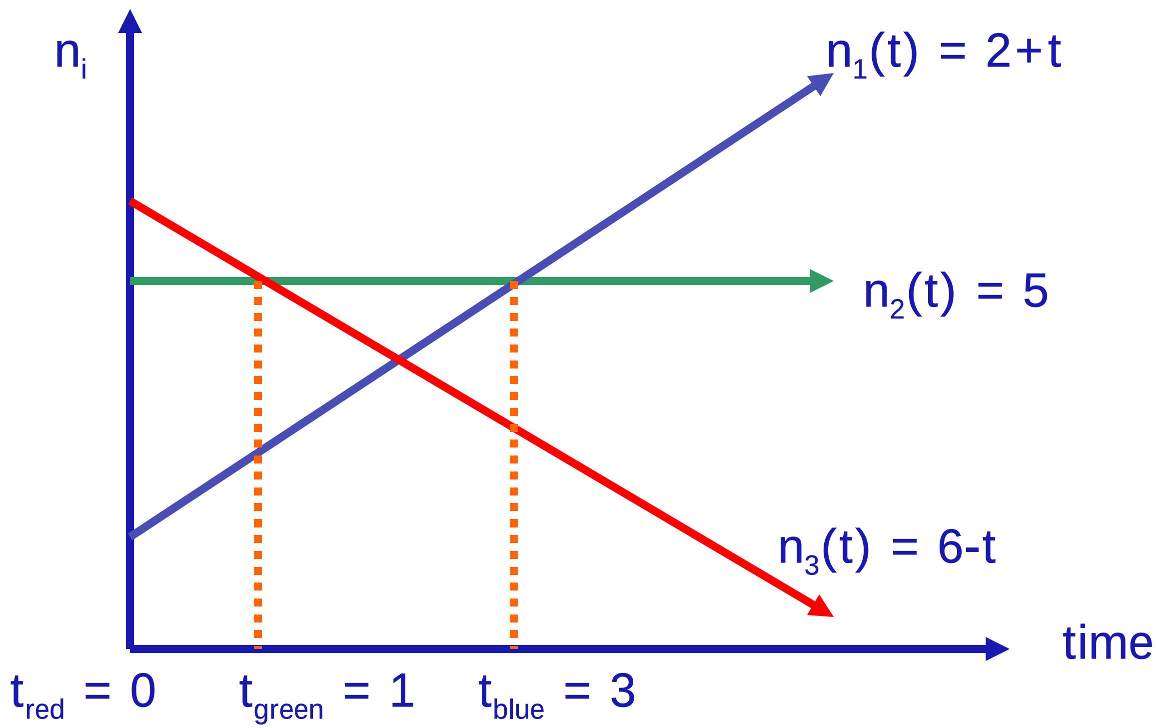
- Just consider upper hull: Finding the maximum

- Consider two algorithms:
  - The March: March through the list
  - The Tournament: pair up the elements and take the max of each pair

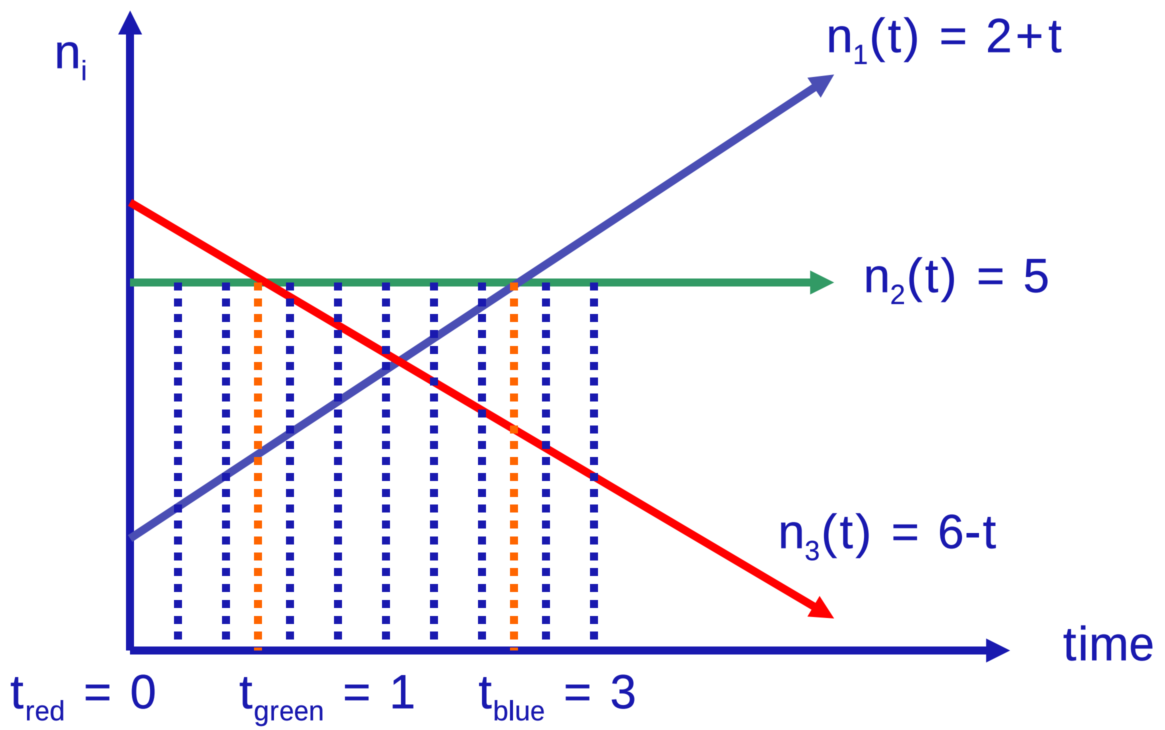


Kinetic Maximum

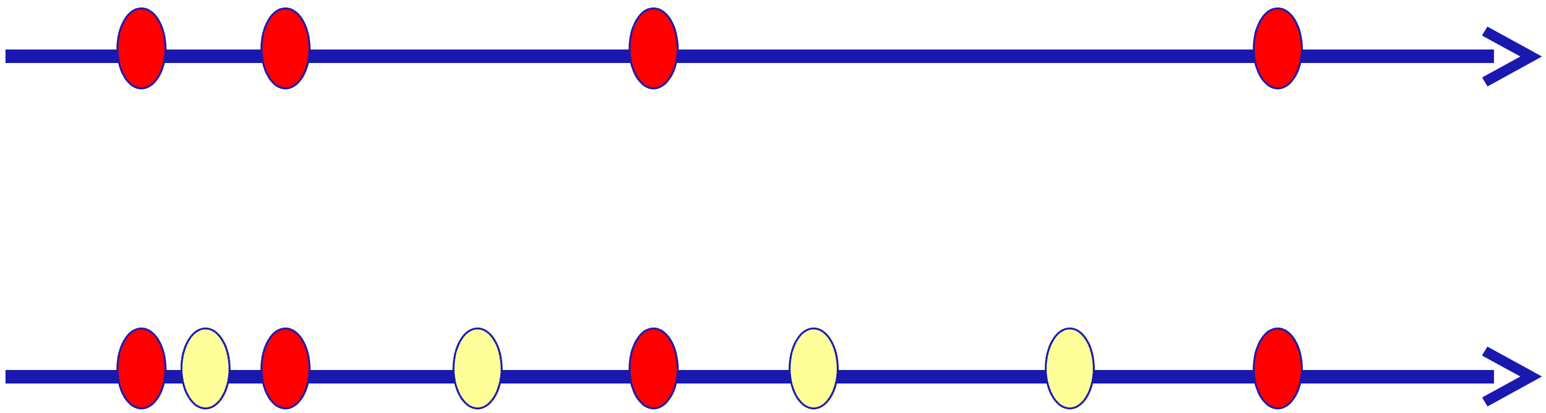
- Numbers increase/decrease continuously in time $n_i(t) = n_i + c_i t$



Sampling



Internal and External Events



External event: Final result changes

Internal event: Final result does not change but a test fails

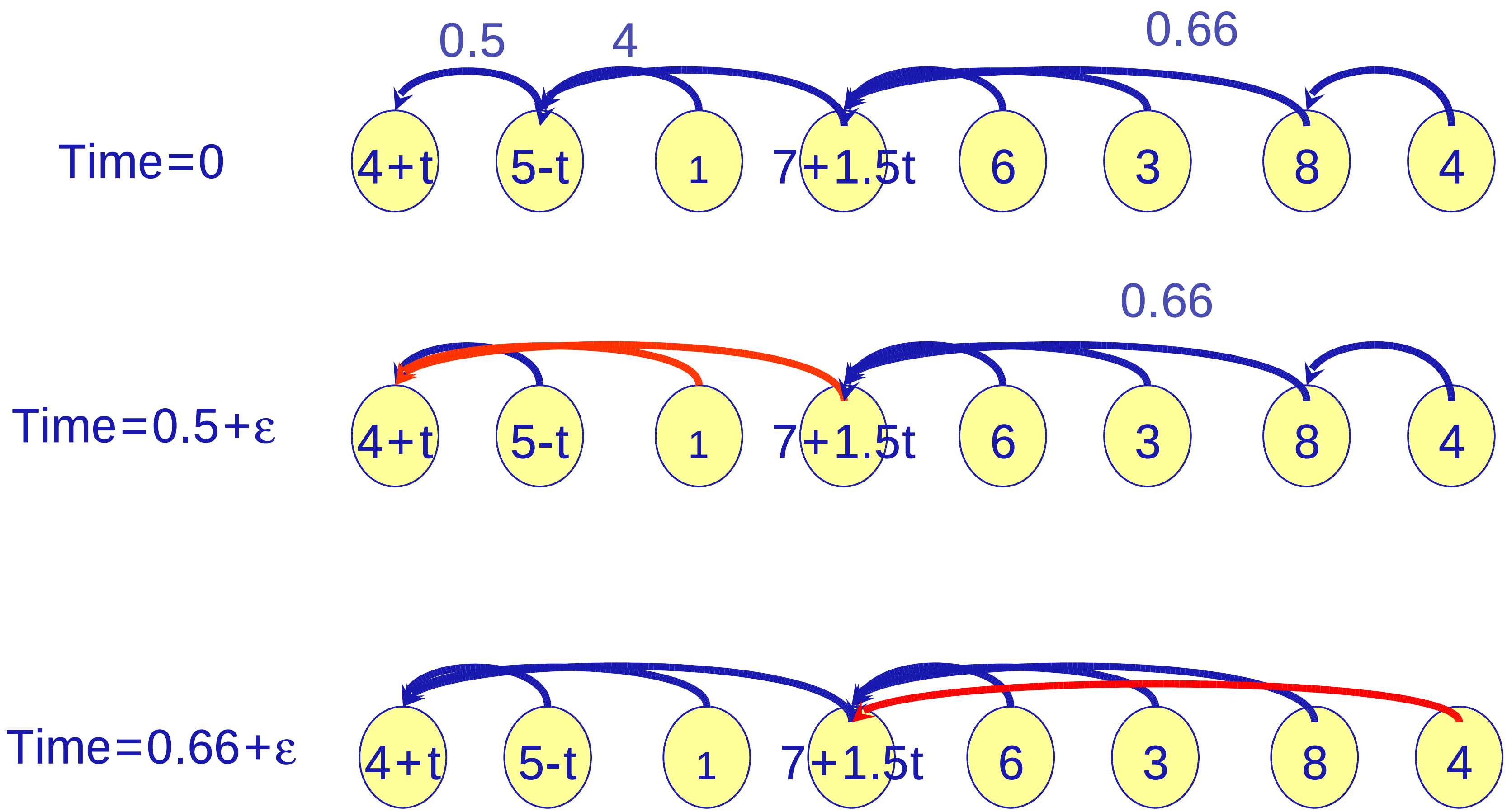
Proof Simulation via Certificates

- A set of comparisons that prove the current maximum
- Associate a **certificate** with each comparison
  - certificate = comparison result + **failure time**
- Consider the times that a certificate fails

- **We need an algorithm that updates the result as well as the certificates**

- **Use adaptivity to obtain to do the update efficiently**

Kinetic March

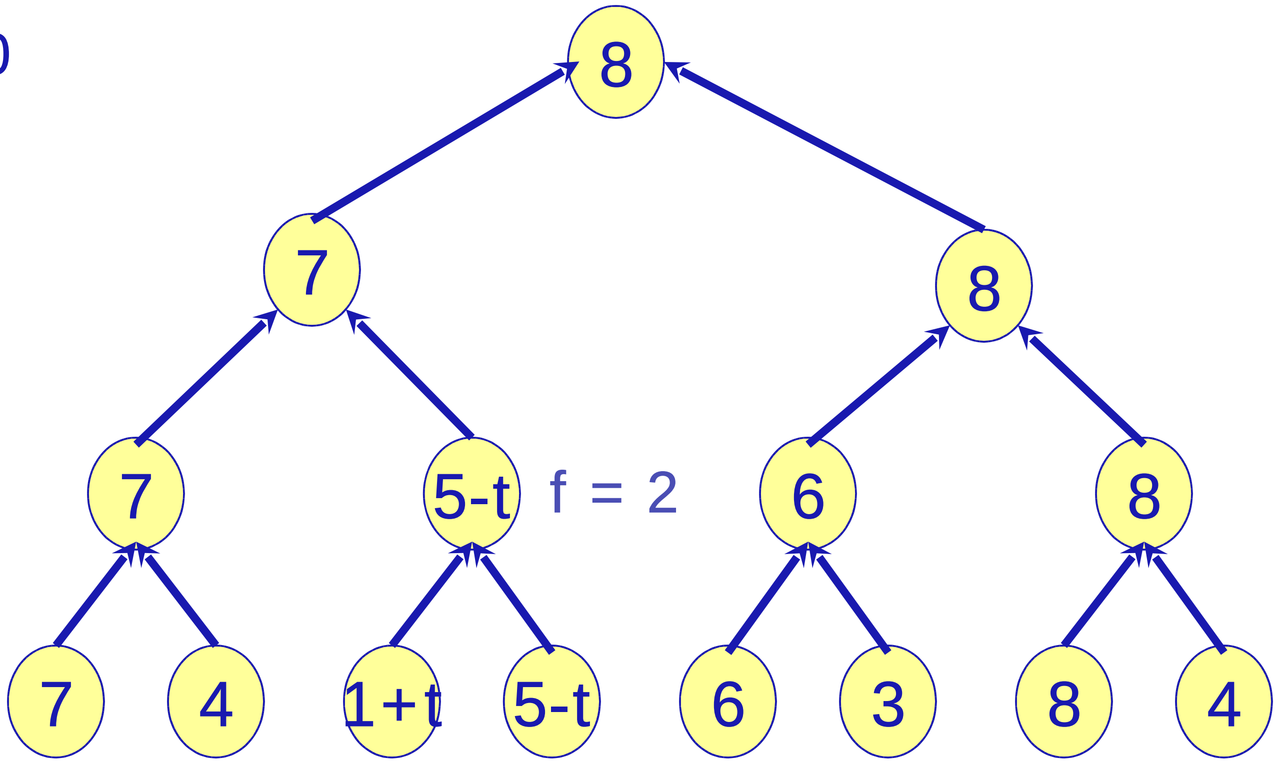


Kinetic March Performance

- O(n): Because an item in the beginning of the list could become the maximum and it will be compared to the rest of the list
- Not acceptable because computing from scratch takes O(n)

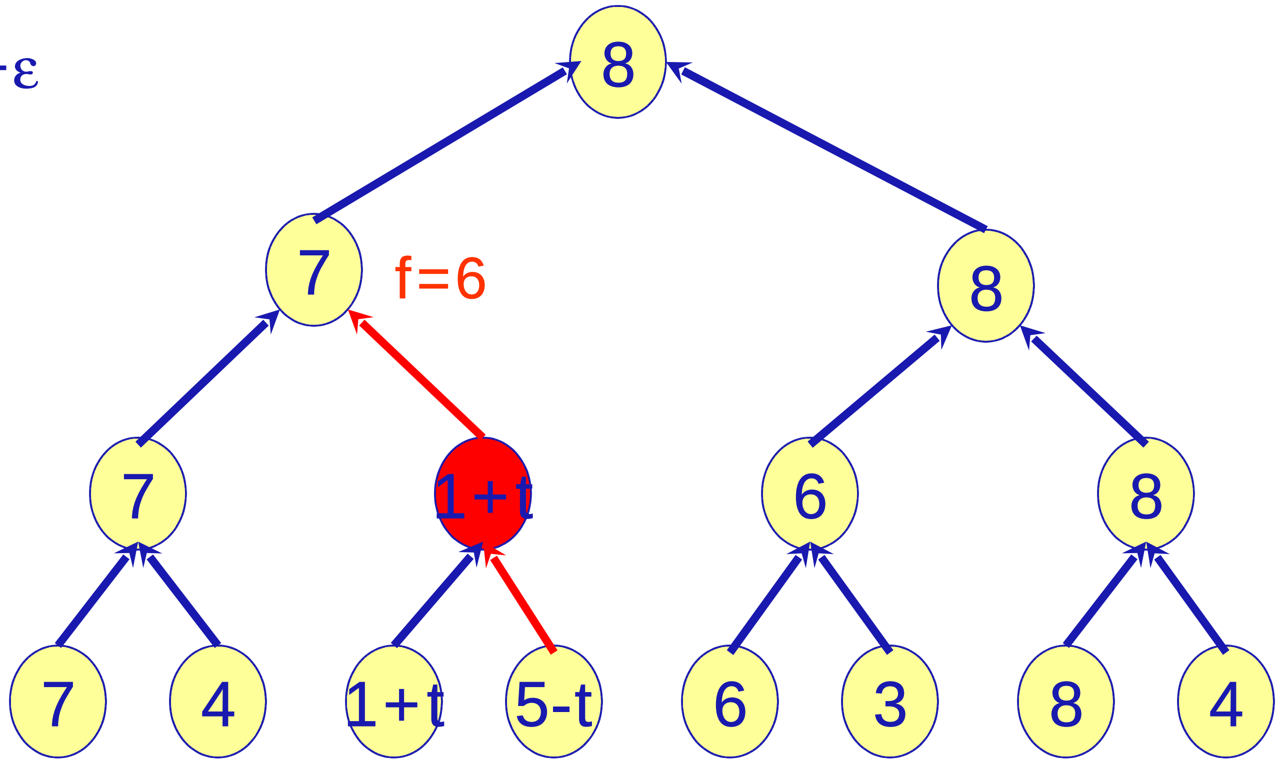
The Kinetic Tournament

Time = 0



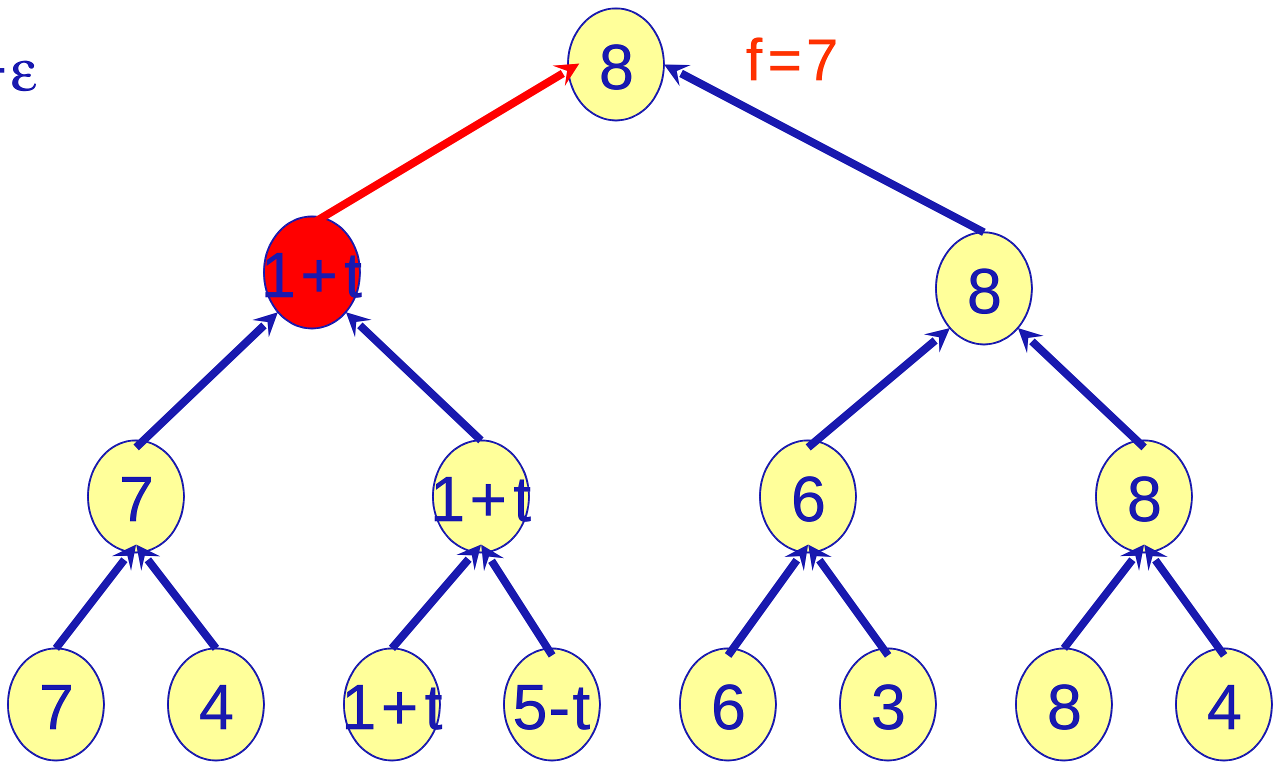
The Kinetic Tournament

Time=2+ε



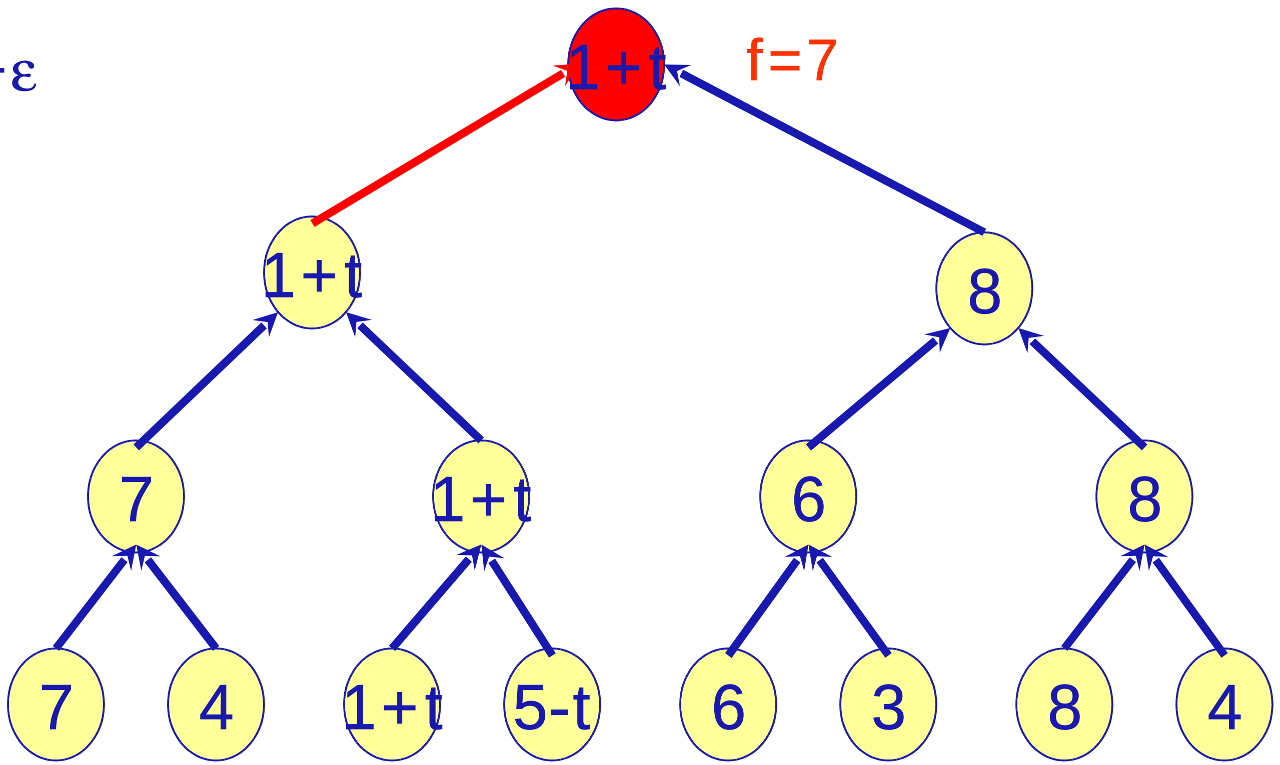
The Kinetic Tournament

Time=6+ε



The Kinetic Tournament

Time=7+ε

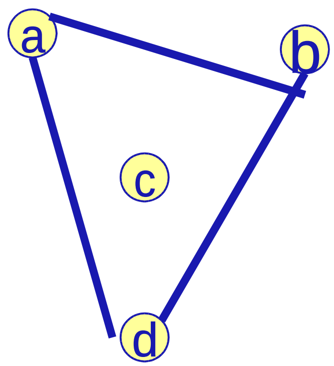


Performance of Kinetic Tournament

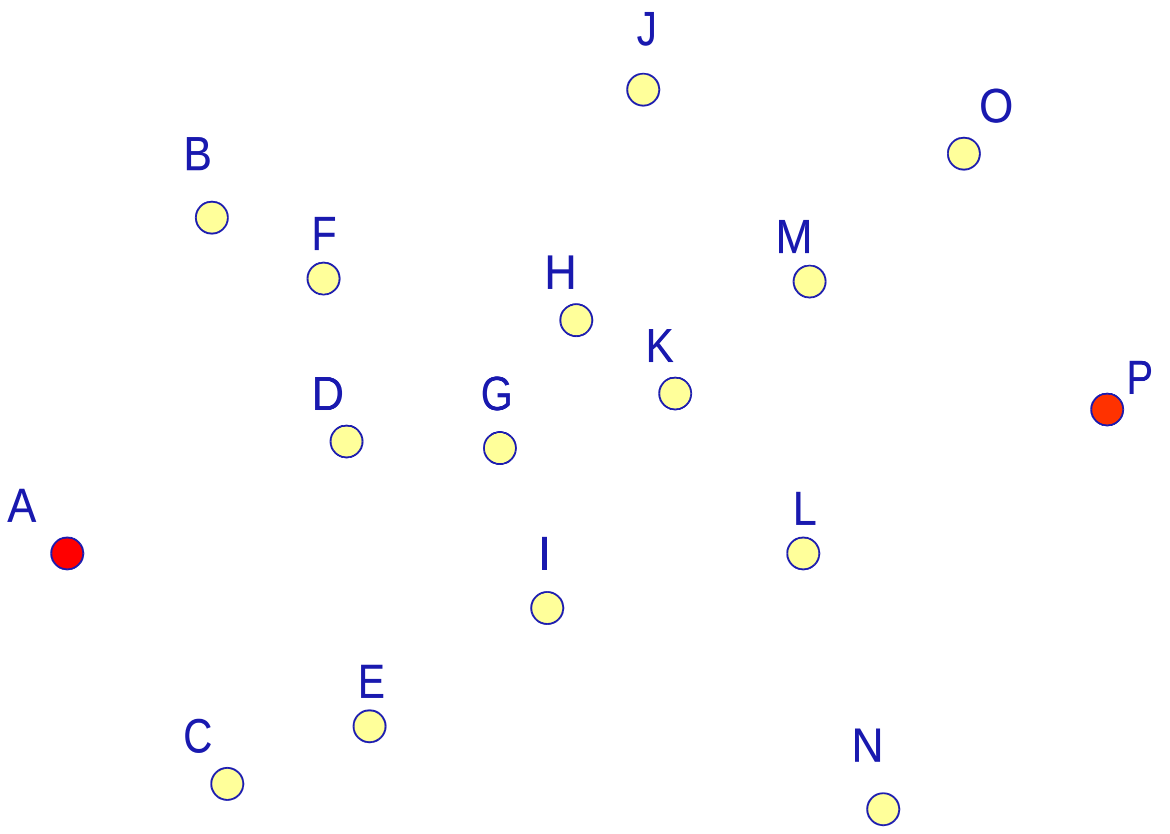
- Worst case log n time per event
- This kinetic algorithm is an adaptive version of the standard tournament algorithm for finding maximum

2-D Convex Hull

- Many algorithms: Quick Hull, Graham Scan, Incremental, Merge Hull, Ultimate, Improved Ultimate…
- We will focus on the Quick Hull algorithm
- Input: A list of points ***P***
- Output: The boundary points on the hull of ***P***
- Example: Input = [a,b,c,d]   Output = [a,b,d]

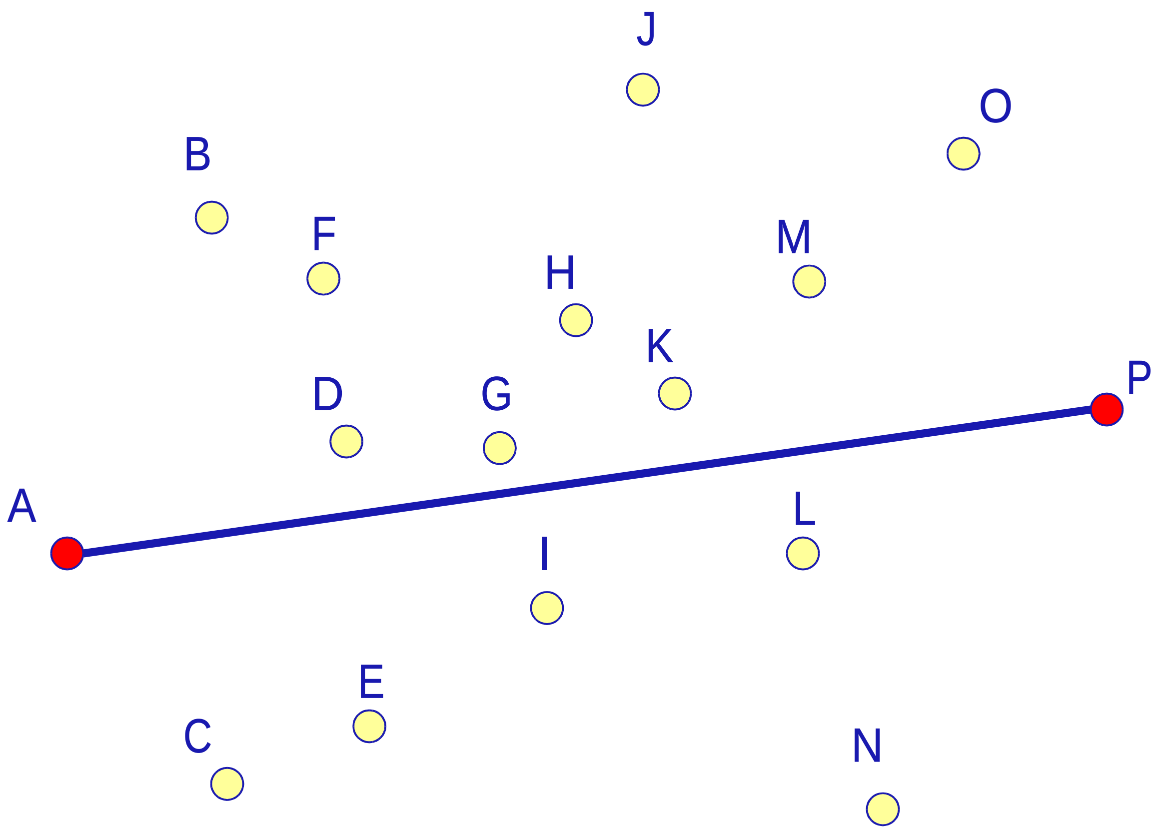


Quick Hull Example



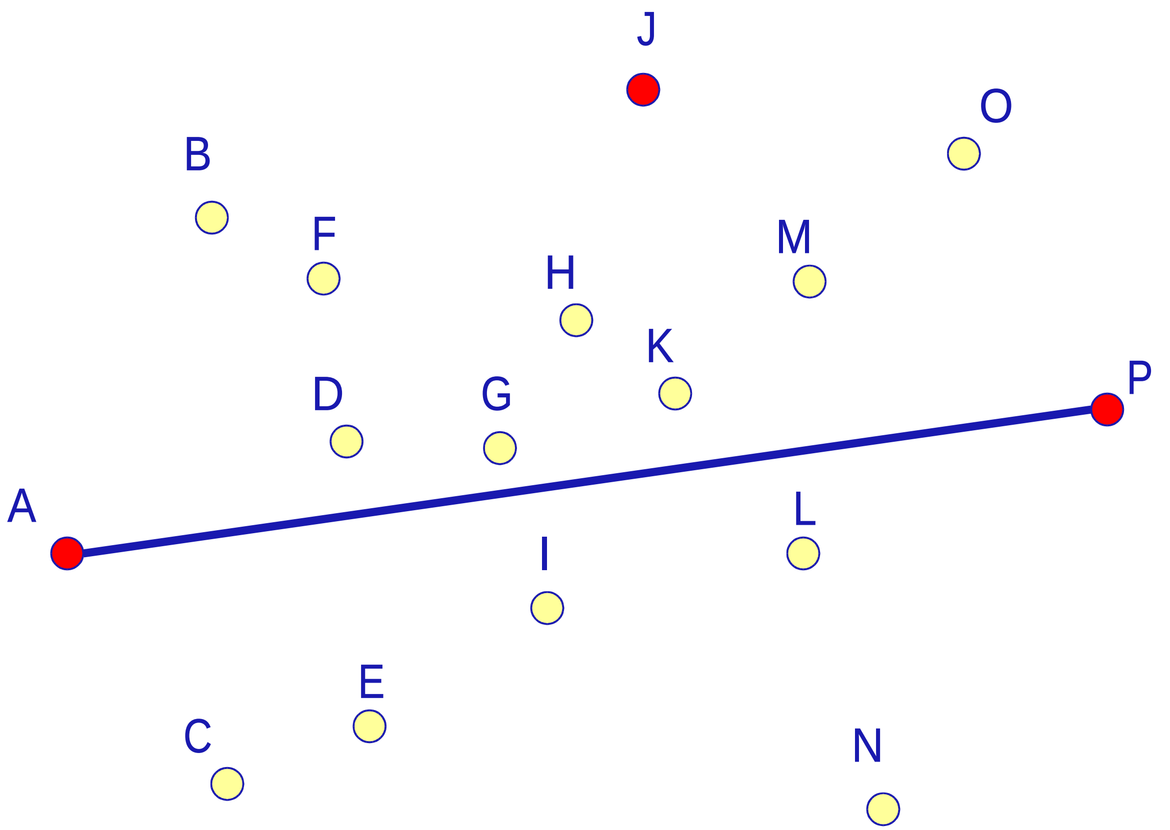
[A B C D E F G H I J K L M N O P]

Quick Hull Example - Filter



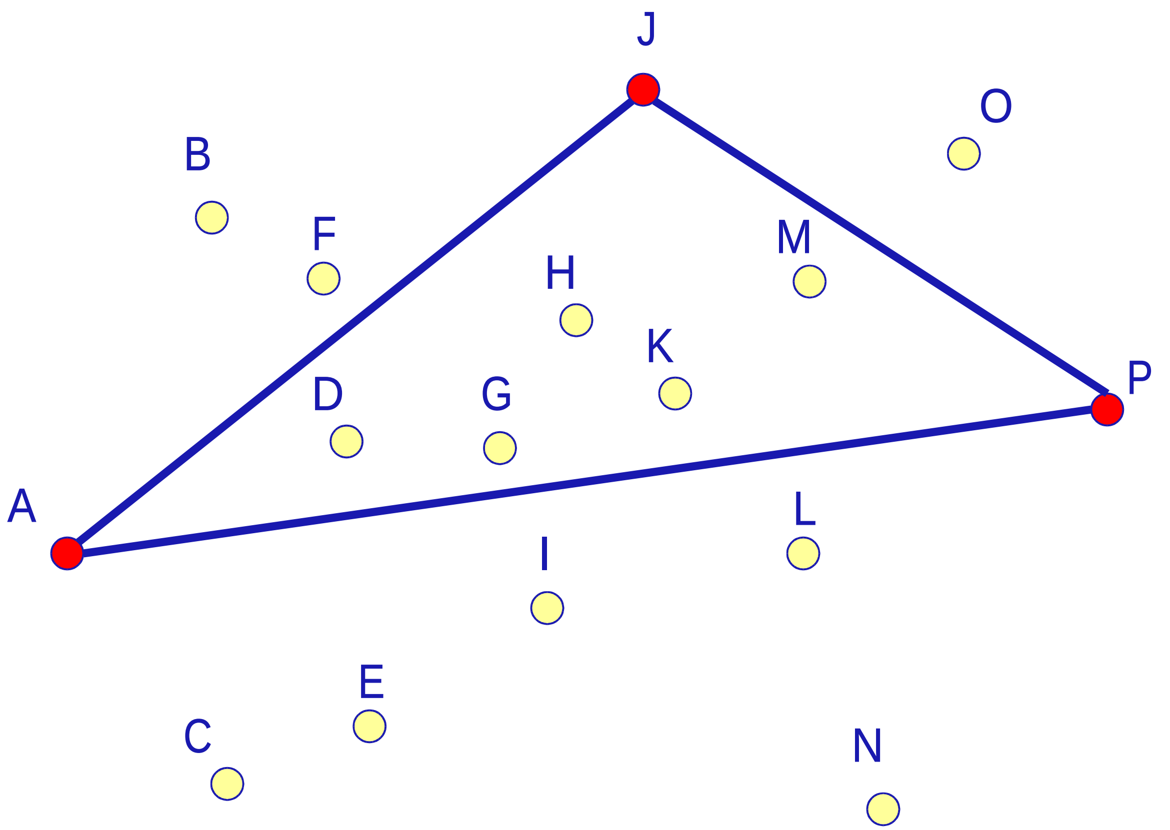
[A B D F G H J K M O P]

Quick Hull Example - Maximum



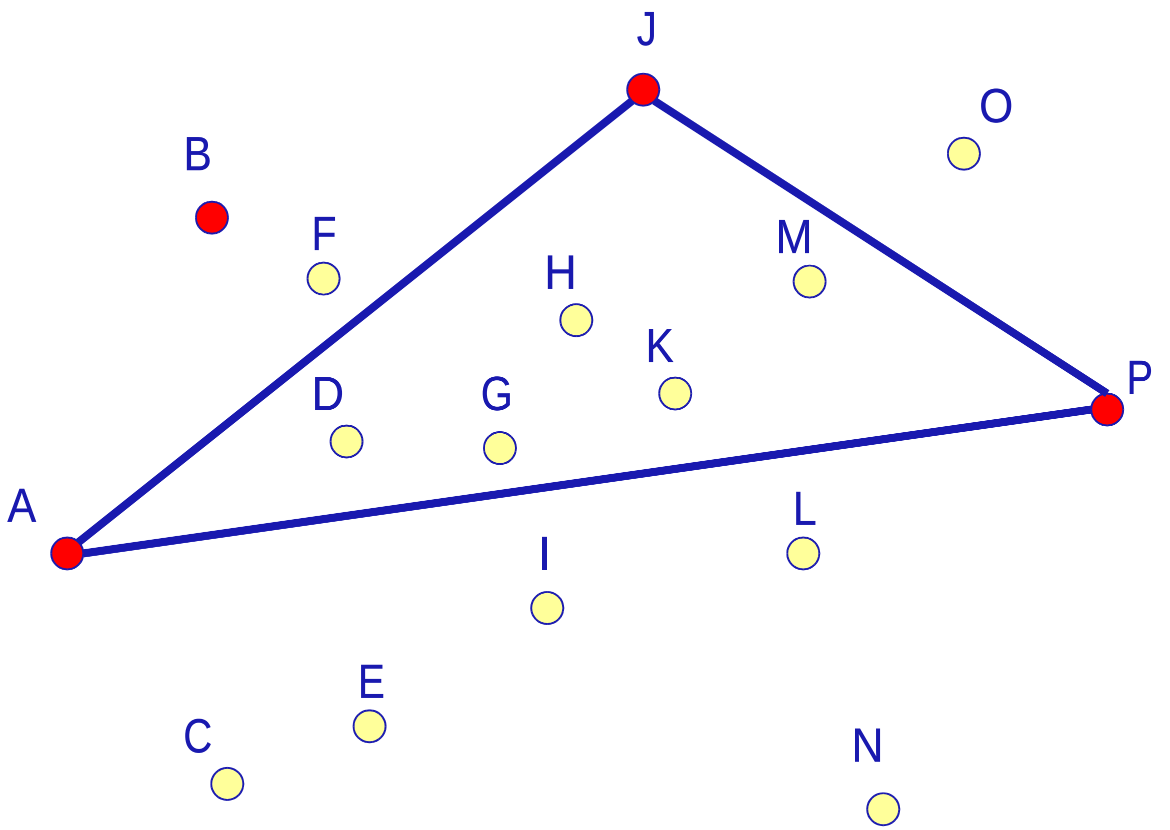
[A B D F G H J K M O P]

Quick Hull Example - Filter



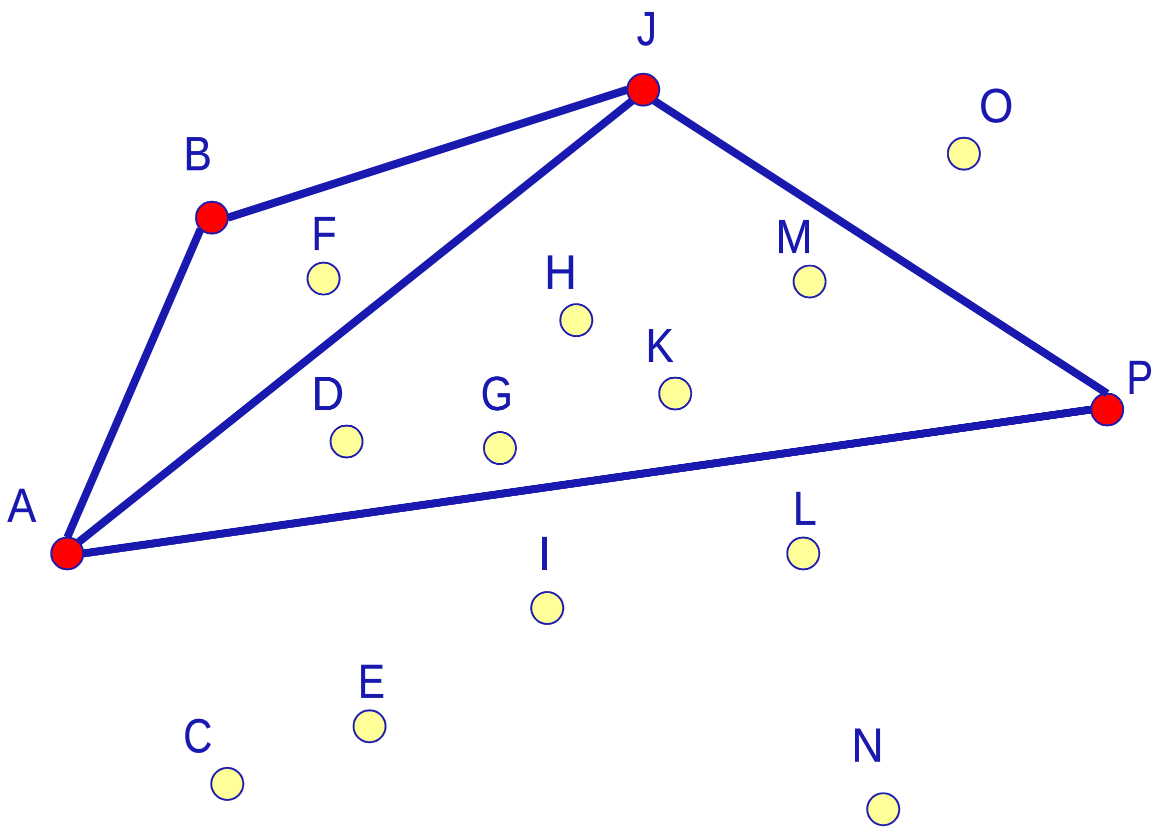
[[A B F J] [J O P]]

Quick Hull Example - Maximum



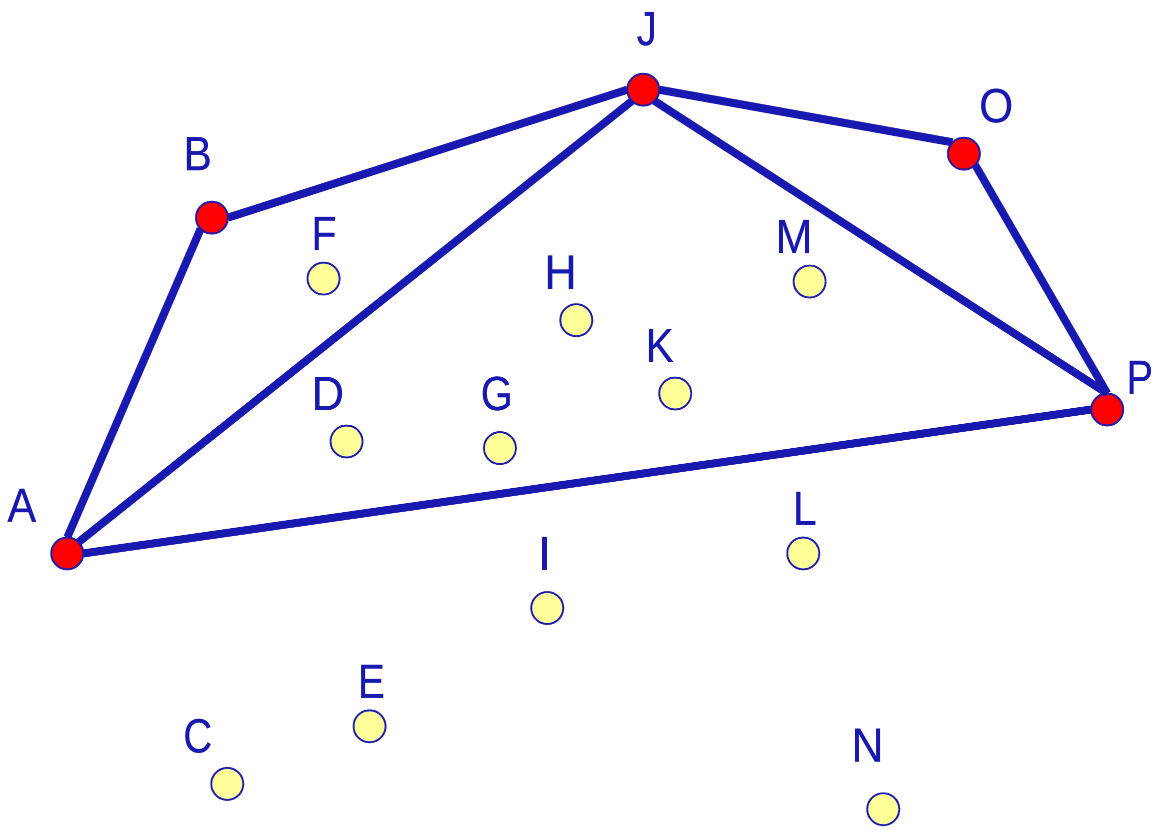
[[A B F J] [J O P]]

Quick Hull Example - Base Case



[[A B] [B J] [J O P]]

Quick Hull Example - Done



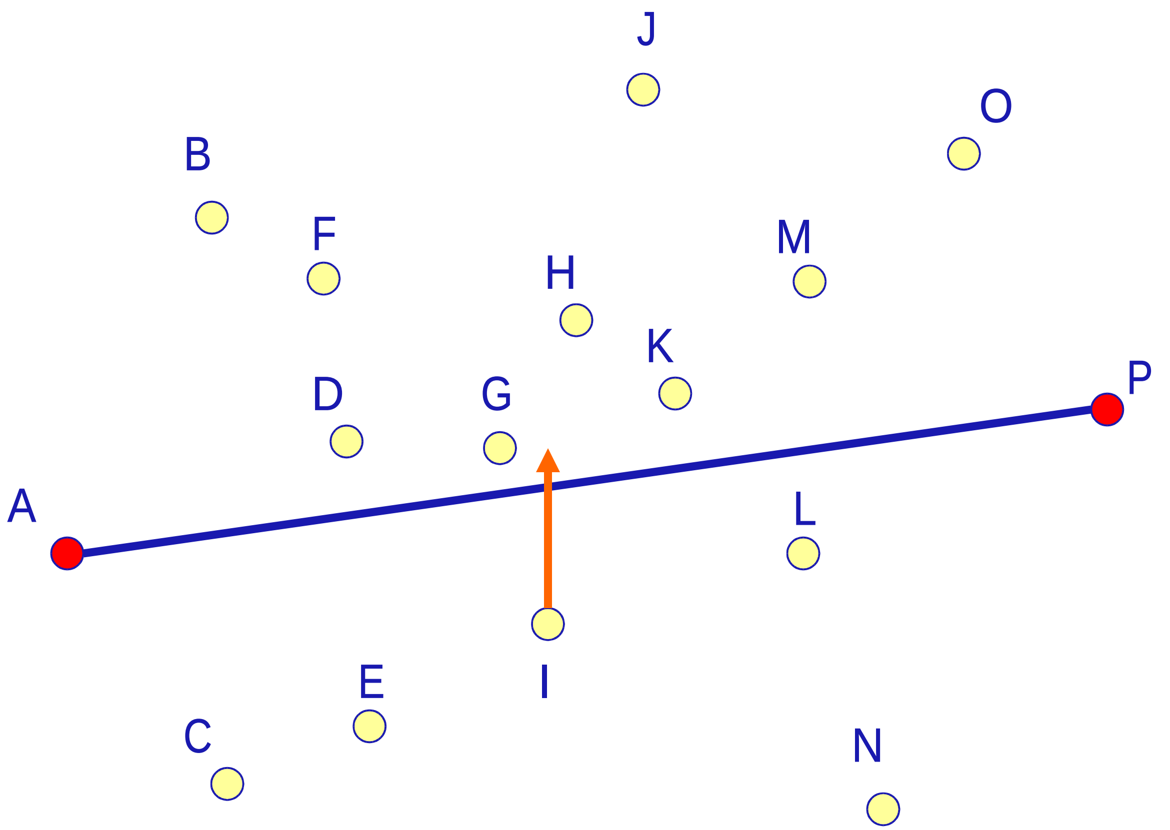
[[A B] [B J] [J O] [O P]]

Kinetic Quick Hull

- Two kinds of tests: Line-side and distance comparisons
- Filtering => Line Side
- Finding the furthest point => Distance comparisons

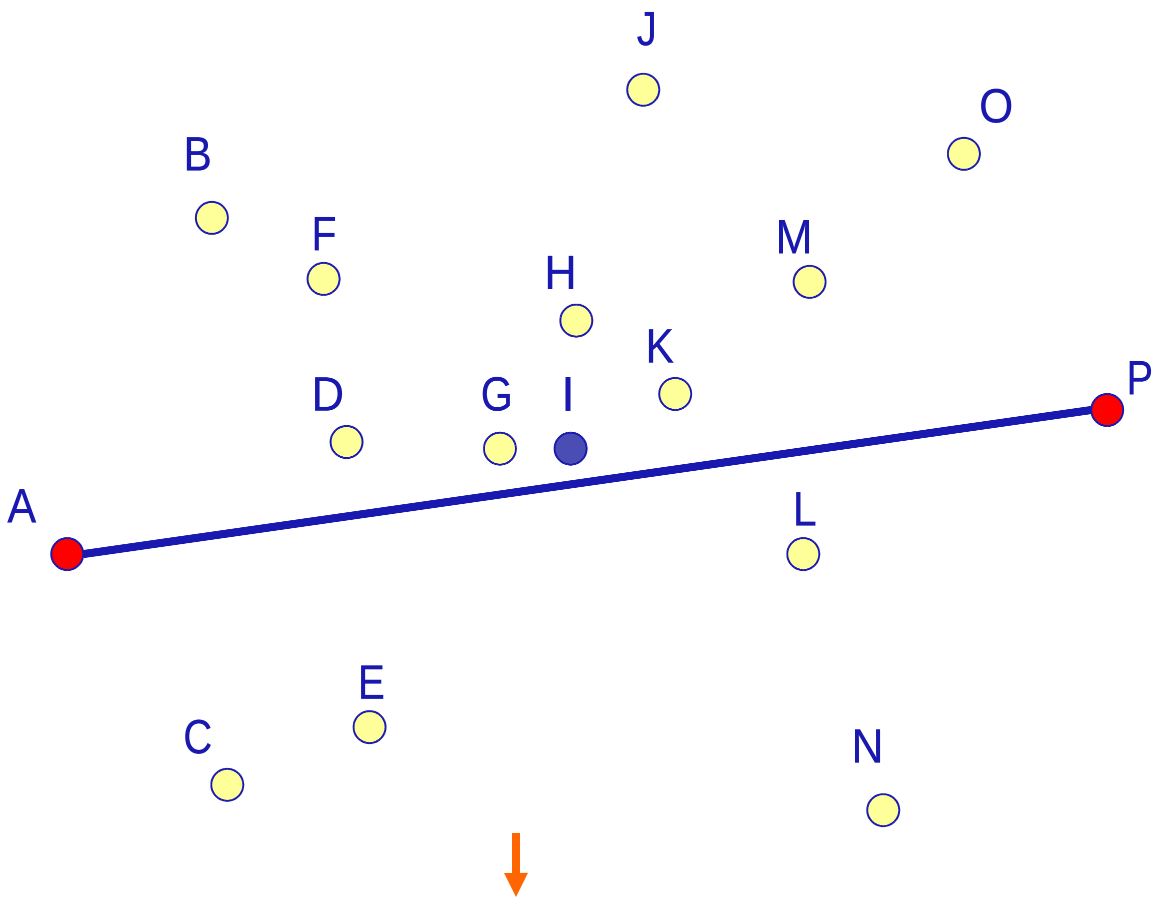
- Have certificates for these two events that is all

Line Side Test Fails



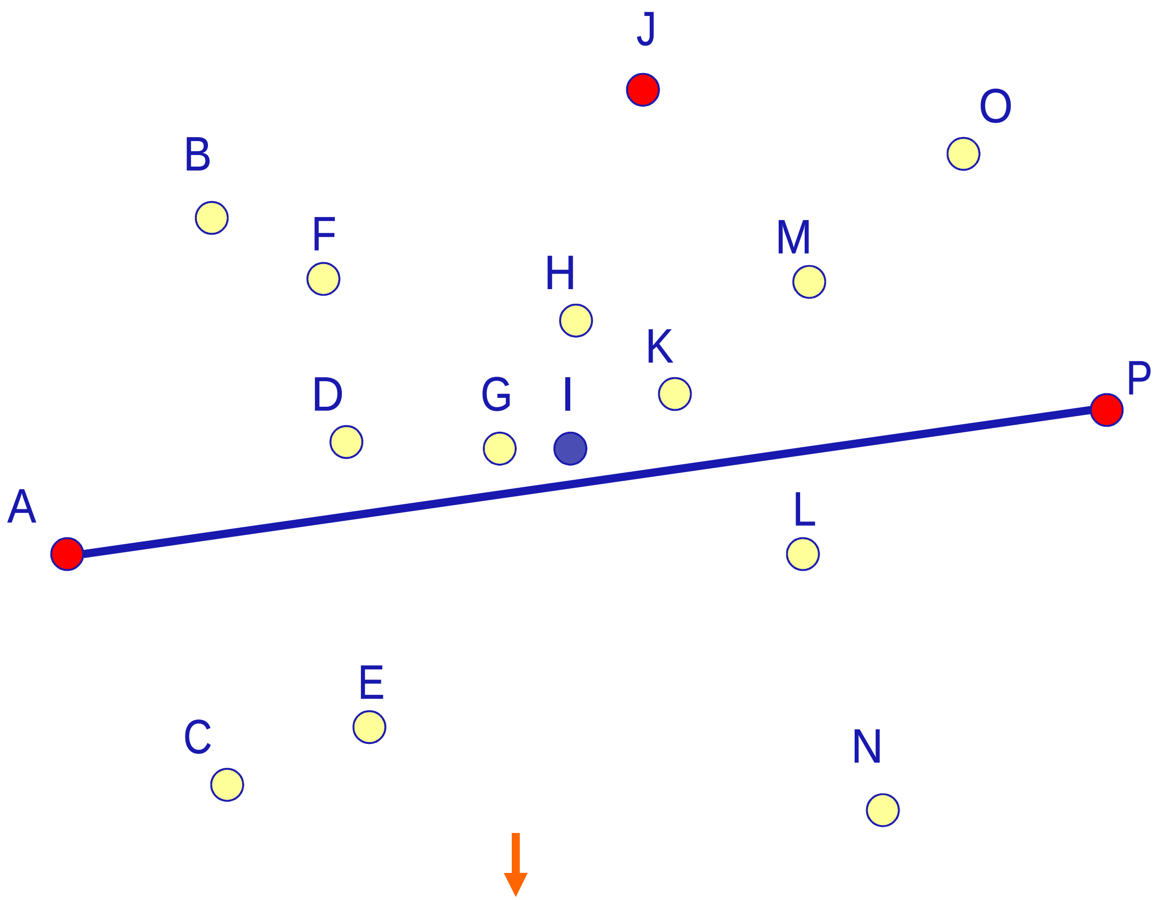
[A B D F G H J K M O P]

“I” is inserted in the middle of the list



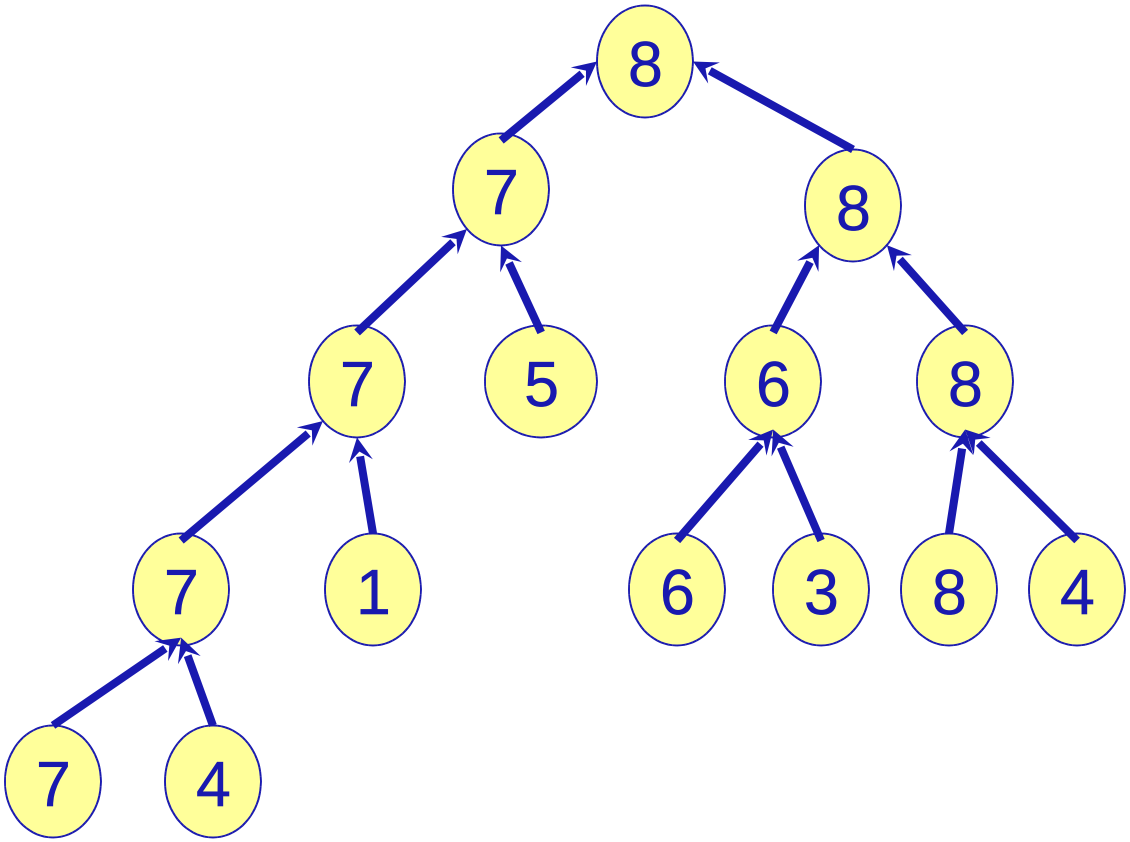
[A B D F G I H J K M O P]

Recompute Maximum

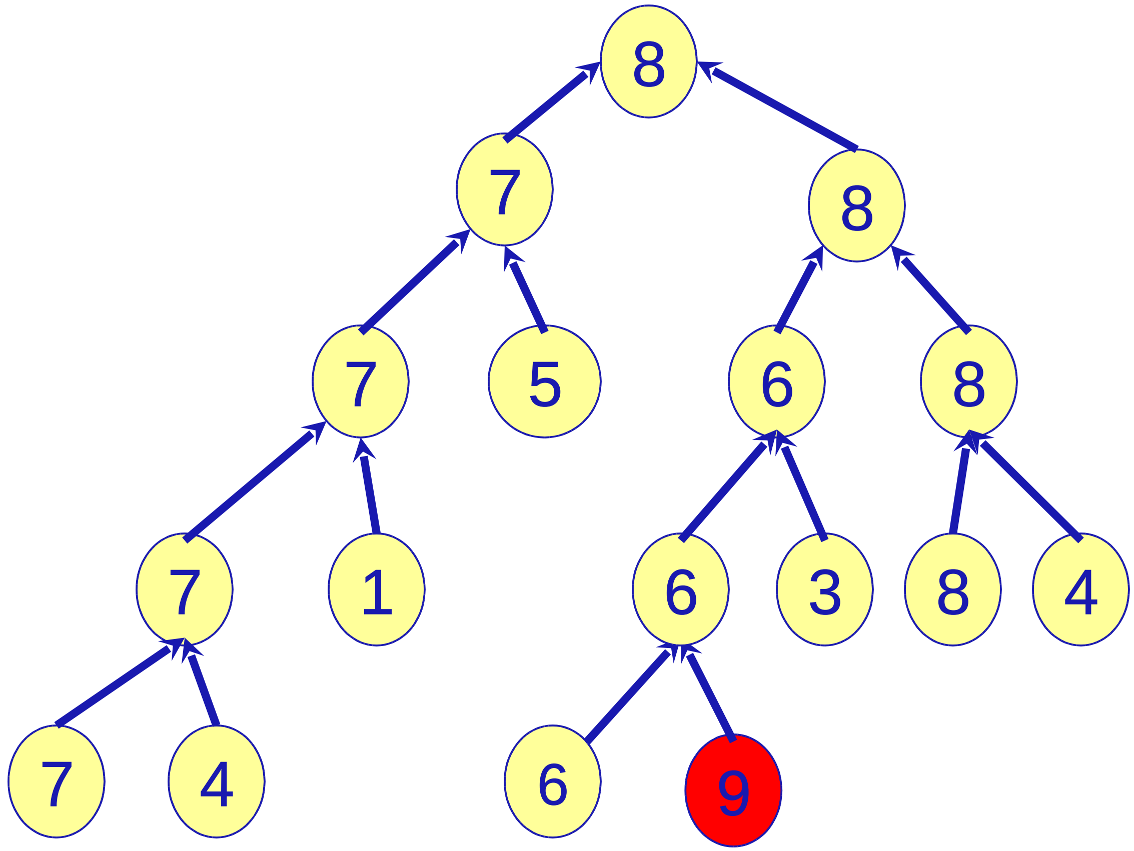


[A B D F G I H J K M O P]

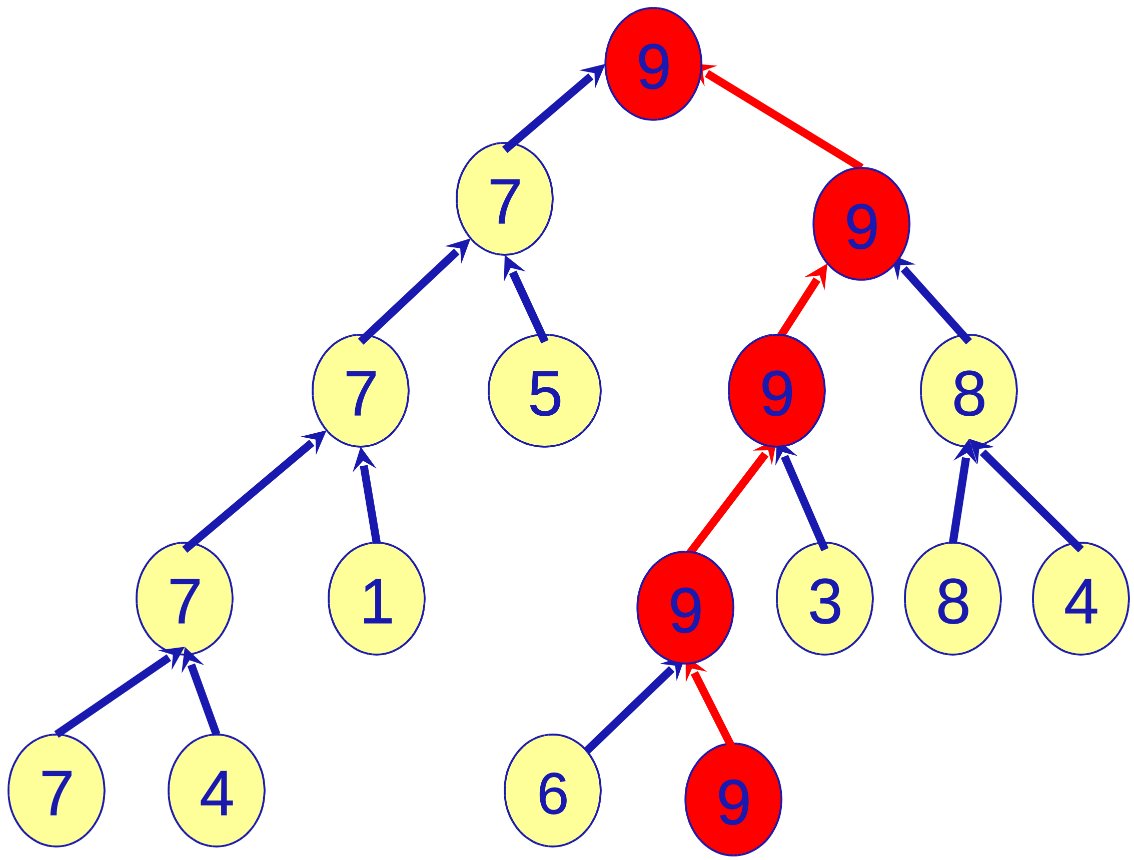
Dynamic Tournament - Random Trees



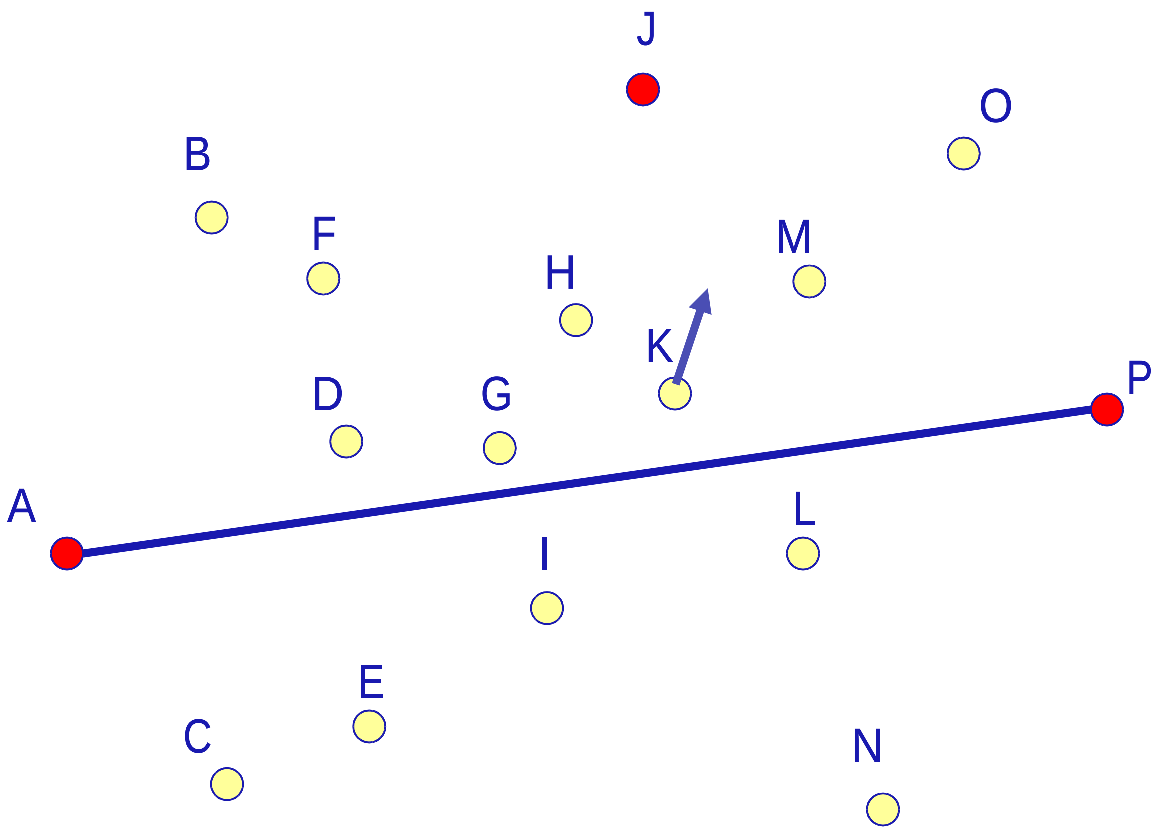
Dynamic Tournament - Random Trees



Dynamic Tournament - Random Trees

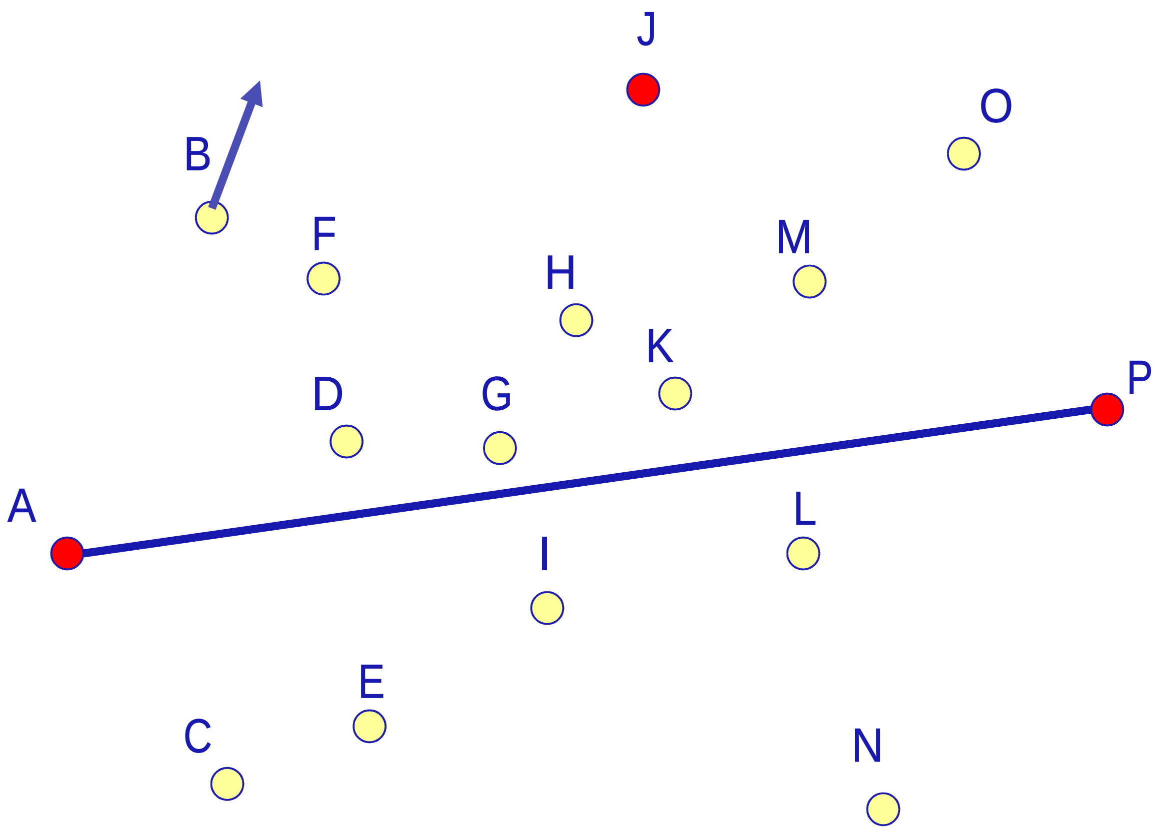


Distance Comparison Fails - Case 1



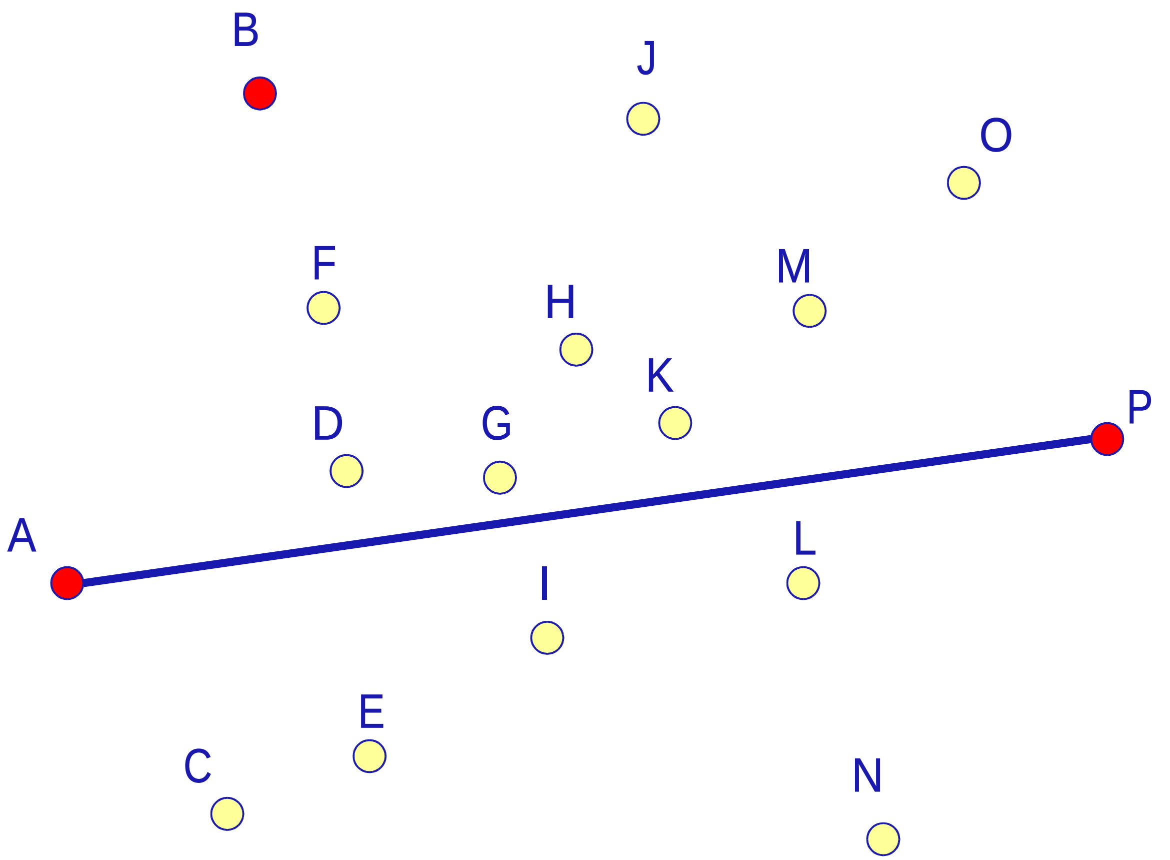
[A B D F G H J K M O P]

Distance Comparison Fails - Case 2



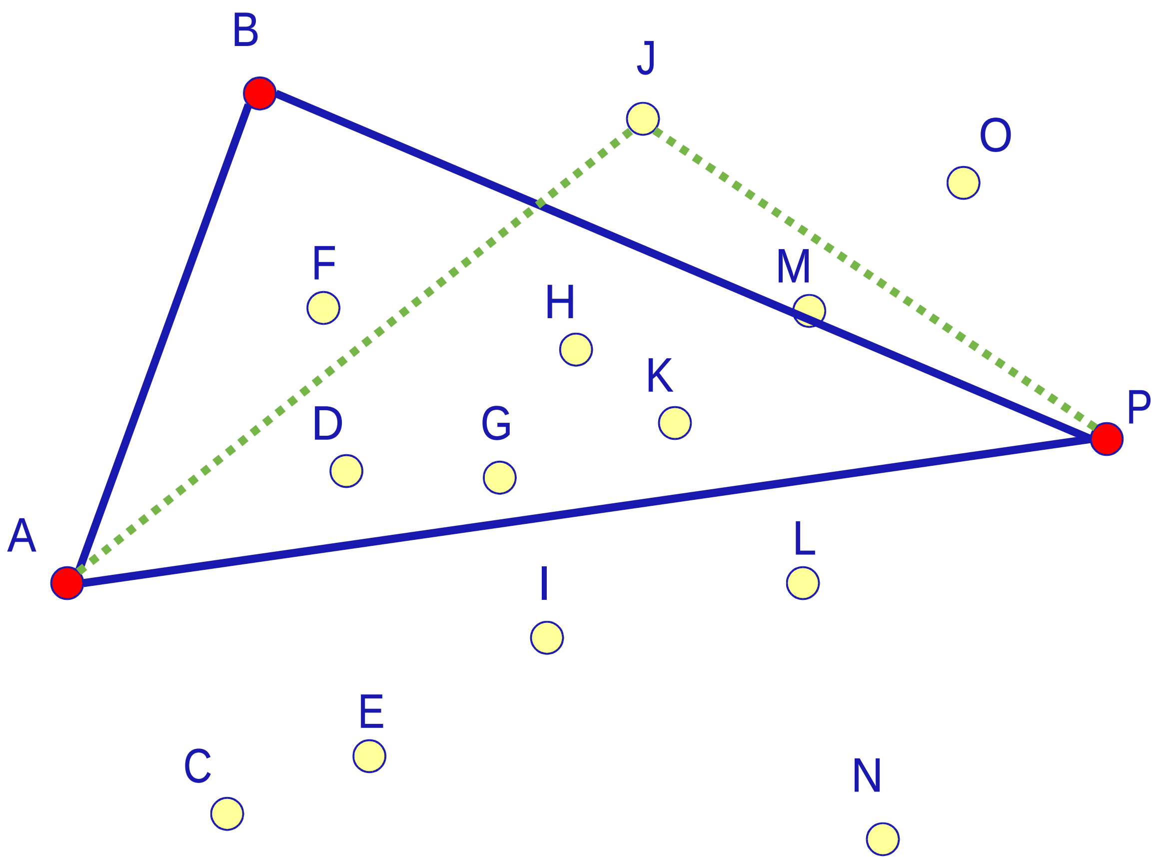
[A B D F G H J K M O P]

“B” is the new maximum



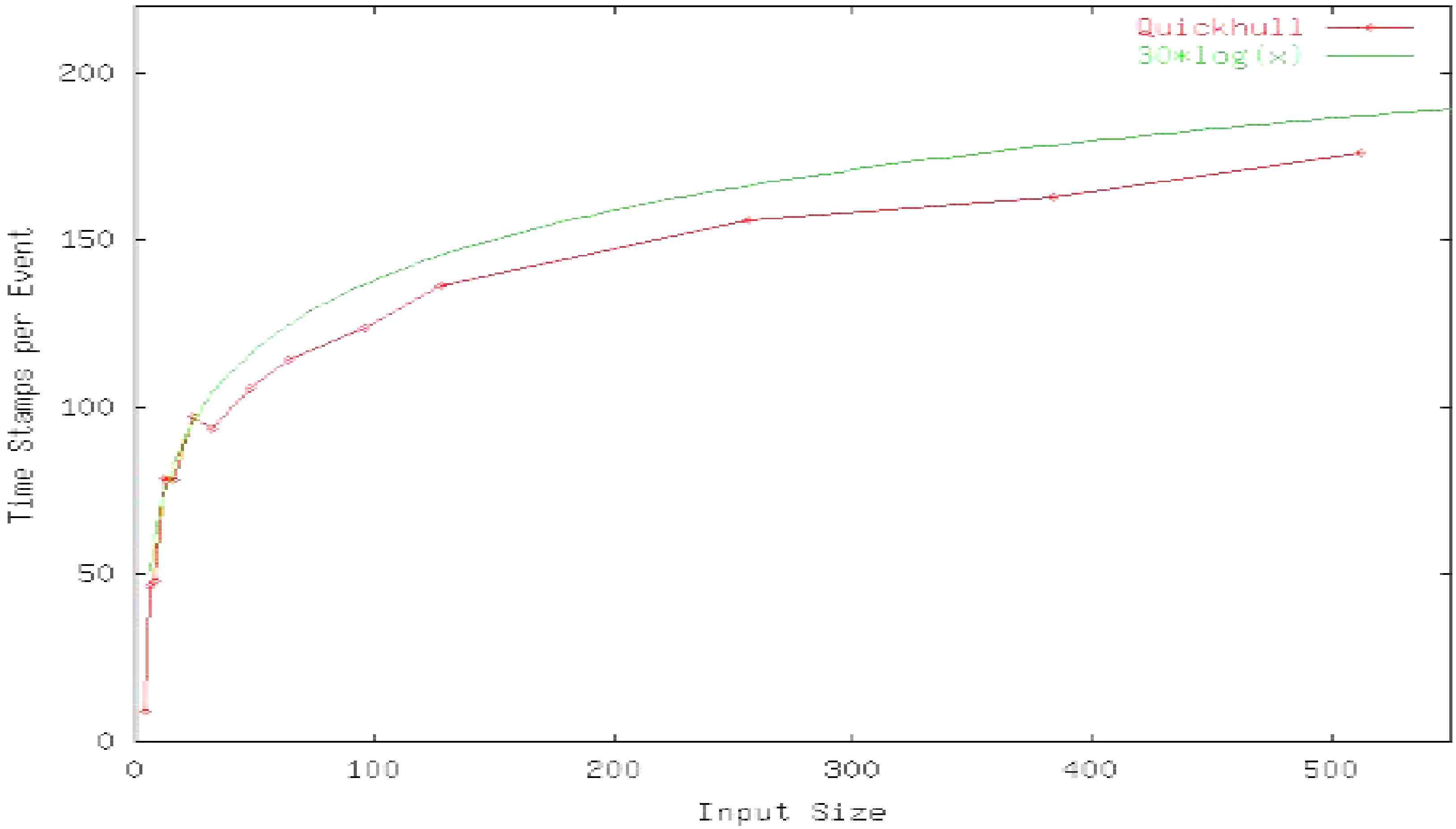
[A B D F G H J K M O P]

New recursive calls



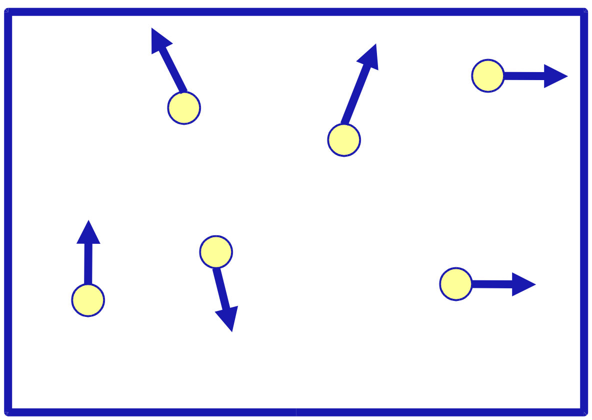
[[A B] [J M O]]

Experiments



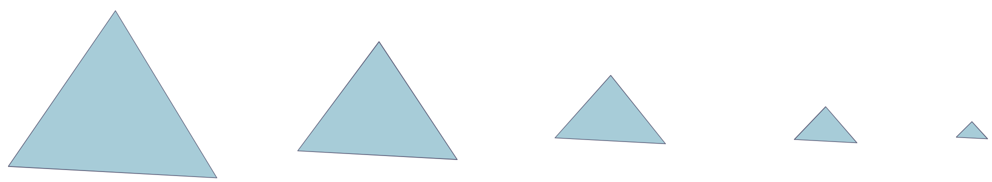
Summary of ConvexHull Work

- Kinetic Algorithms for convex hulls using adaptivity
  - Timothy Chan's *O(h log n)* algorithm: Improved "Ultimate Convex Hull": Have a working version
  - QuickHull
- Bounce events: Can maintain convex hull of points in a box - the points bounce off of the walls of the box
- Streamlined library for kinetic convex hulls in the SML language
  - A standard algorithm can be made kinetic in a few hours of work



Parallel Tree Contraction

- Fundamental technique [Miller & Reif '85]
- Contraction proceeeds in rounds
  - Each round shrinks the tree by a constant factor
  - Expected ***O(logn)*** rounds

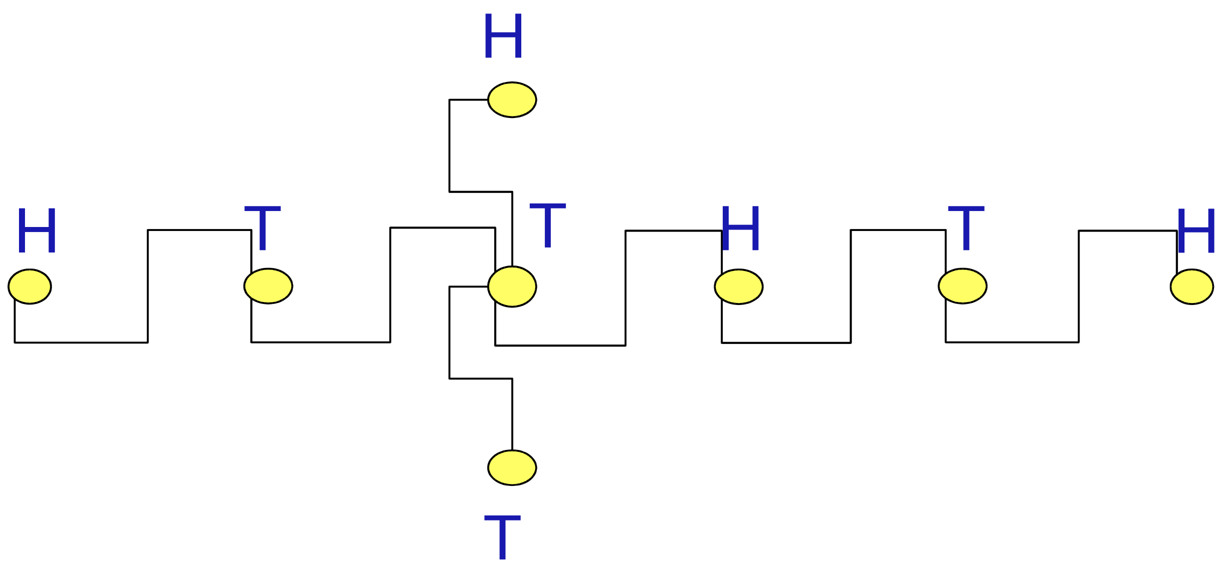


- Innovative Idea: Shrink the tree by local operations

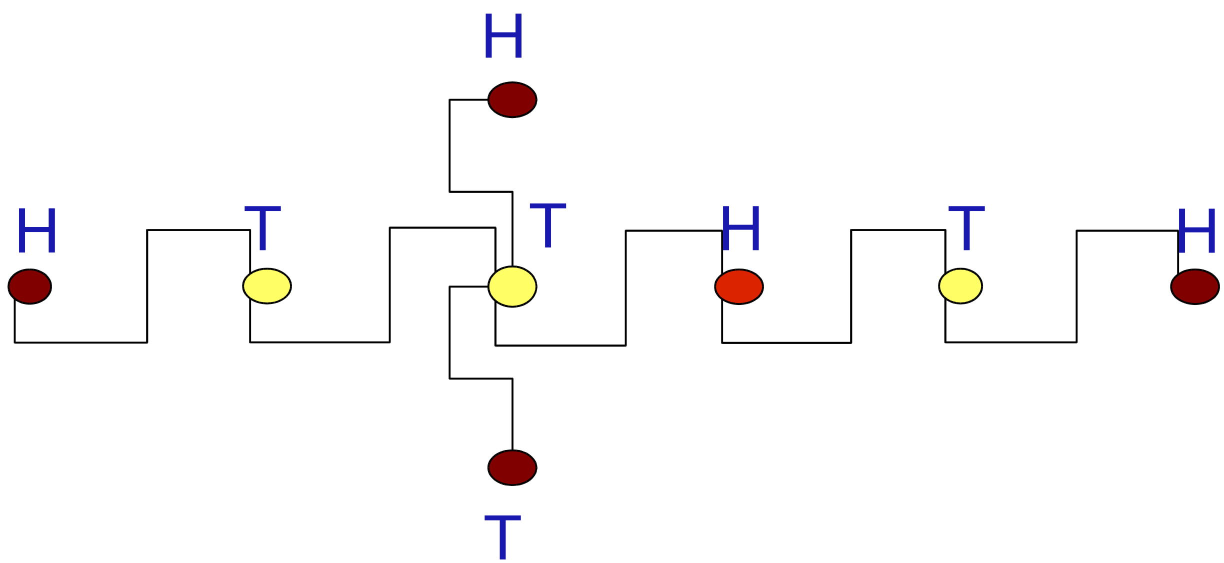
Parallel Tree Contraction

- Start with a tree
- In each round:
  - Each node flips a coin
  - If leaf node then rake
  - If degree=2 and flip = H, and neighbors = T then contract
- Expected ***O(logn)*** rounds.

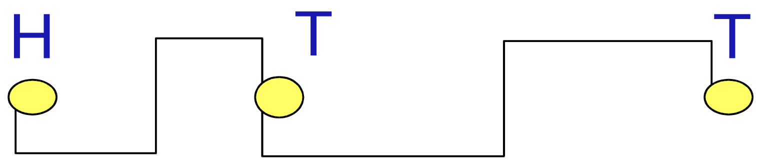
Example



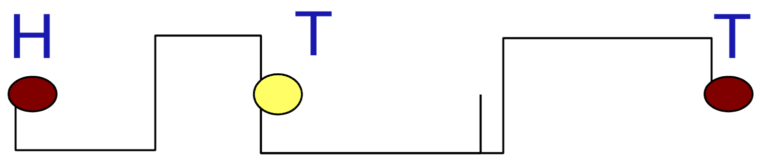
Contracting and Raking



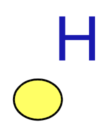
Contracting and Raking



Contracting and Raking (cont.)



Done



Dynamic Trees Problem

- Given a forest of weighted trees
- Operations
  1. **`Link:`** edge insertion
  2. **`cut:`** edge deletion
  3. Queries
     - Heaviest edge in a subtree?
     - Heaviest edge on a path?

Data Structures for Dynamic Trees

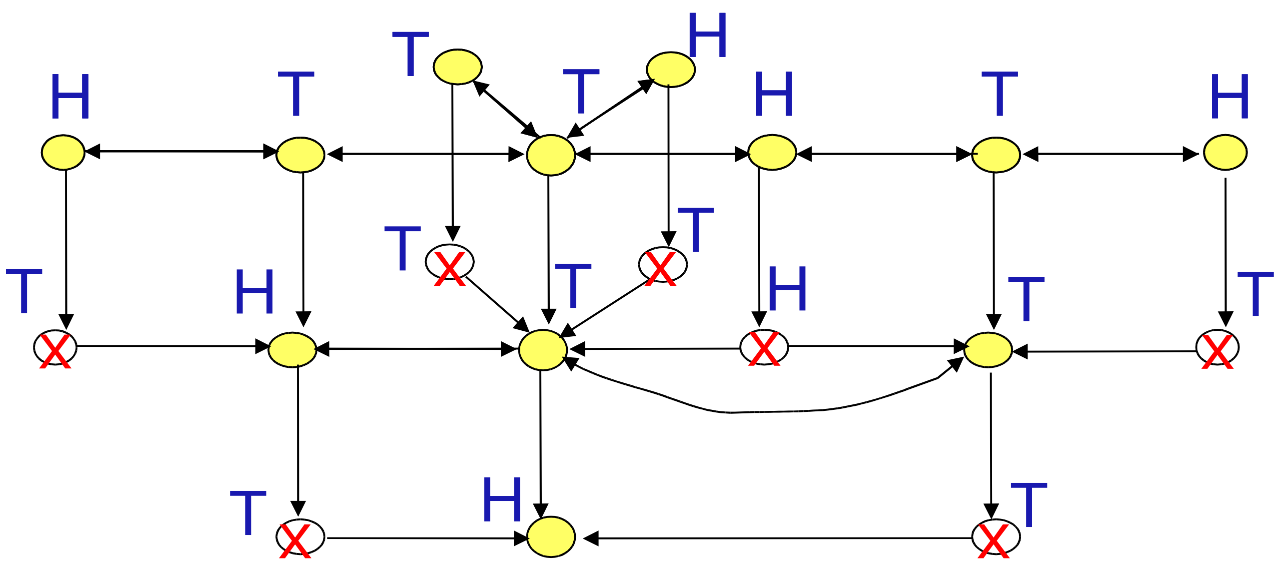
- Sleator Tarjan '85
  - Amortized ***O(logn)*** and worst-case ***O(logn)***
- Topology Trees [Frederickson '93]
  - Ternary (degree-tree) trees
  - Worst case ***O(logn)***
- Top Trees [AlstrupHoLiTh '97]
  - Generalize Topology Trees for arbitrary degree

- Idea: Trees as paths

Dynamic Parallel Tree Contraction

- Keep a copy of each round of the initial run.
- Each round affects next round.
- The nodes that “live” to the next round copy their neighbors scars, and pointers to them.
- Dependencies are based on what the node reads to do its work.

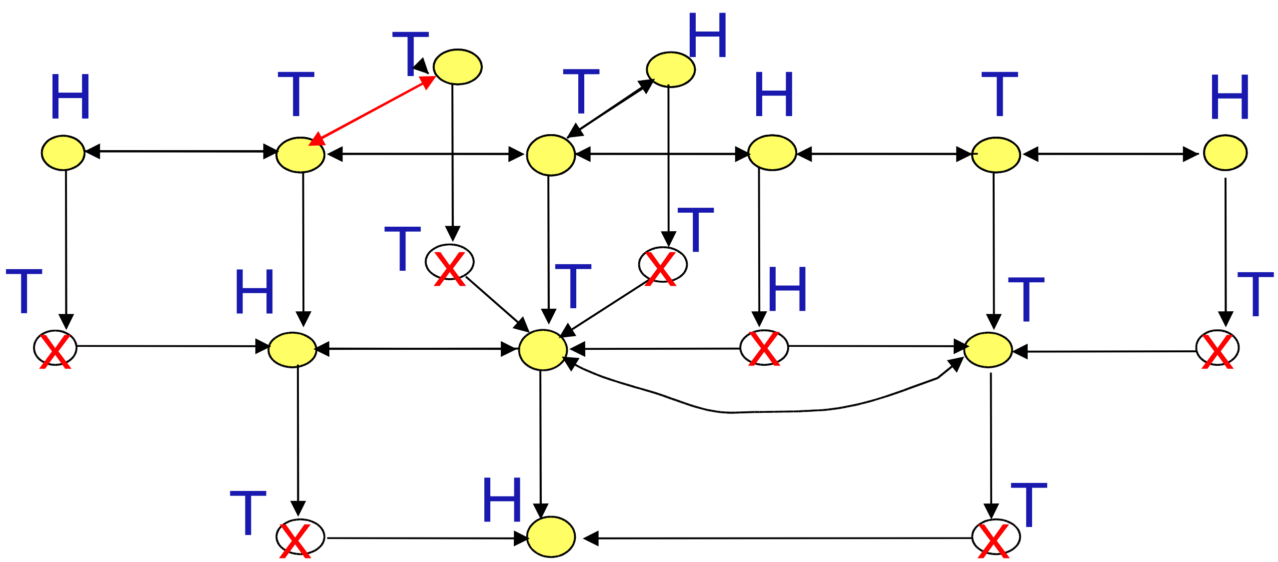
Dynamic Parallel Tree Contraction



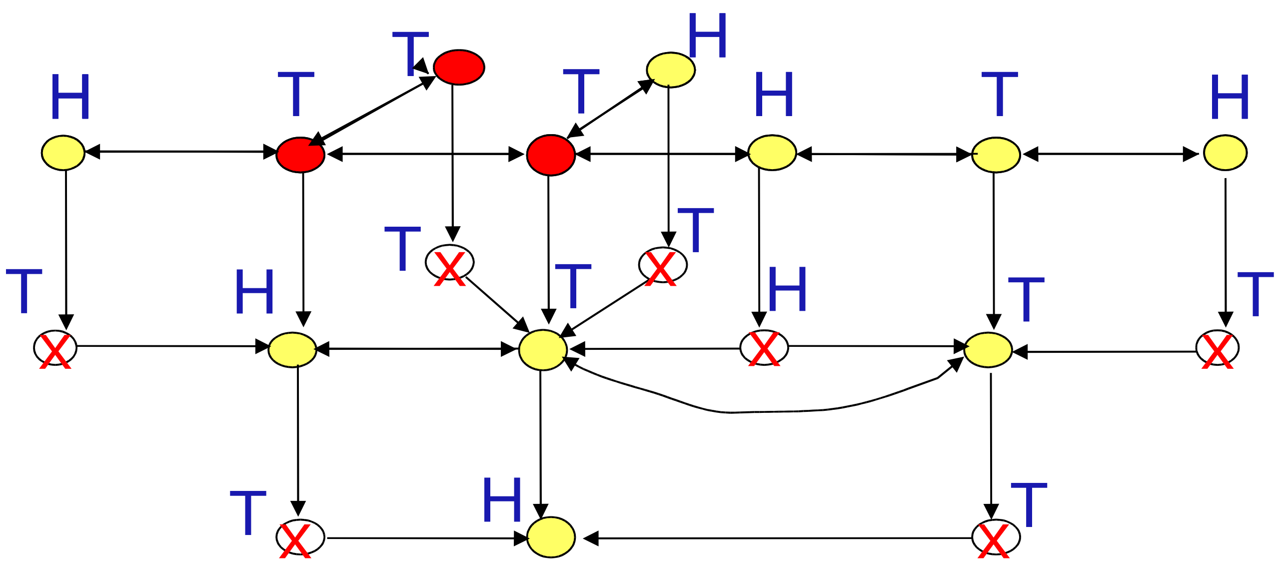
Propagation

- If any data changes nodes whose action depend on that data are woken up.
- Wake-up only those nodes that get affected by a change.
- Run same code as in original run.
- Expected constant amount of nodes woken up per round.

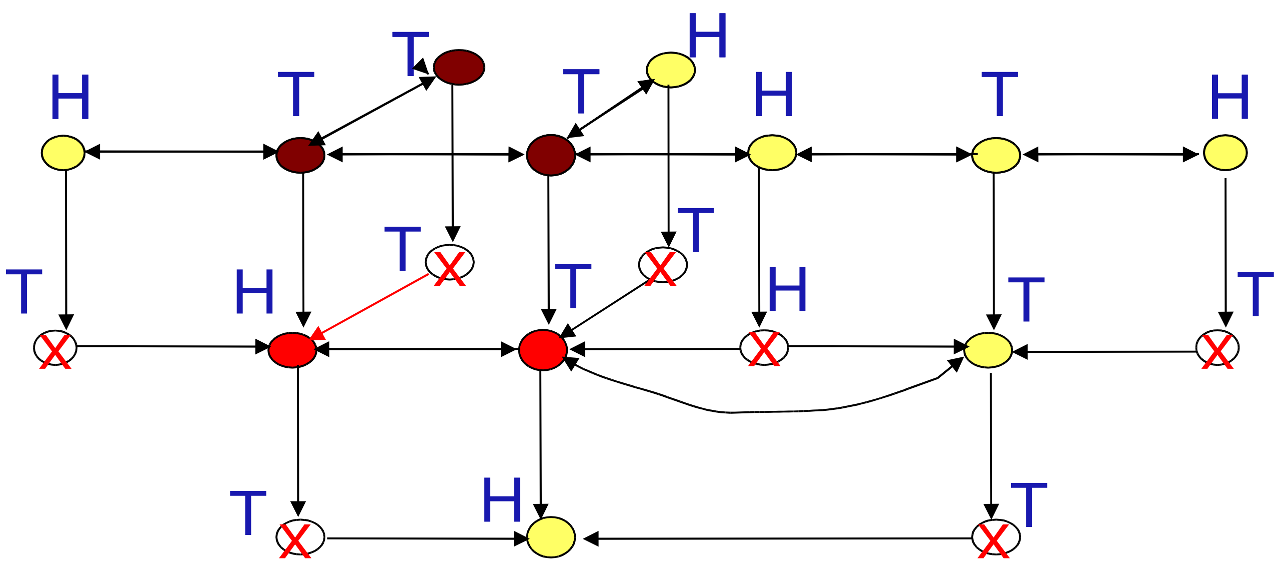
Change an edge



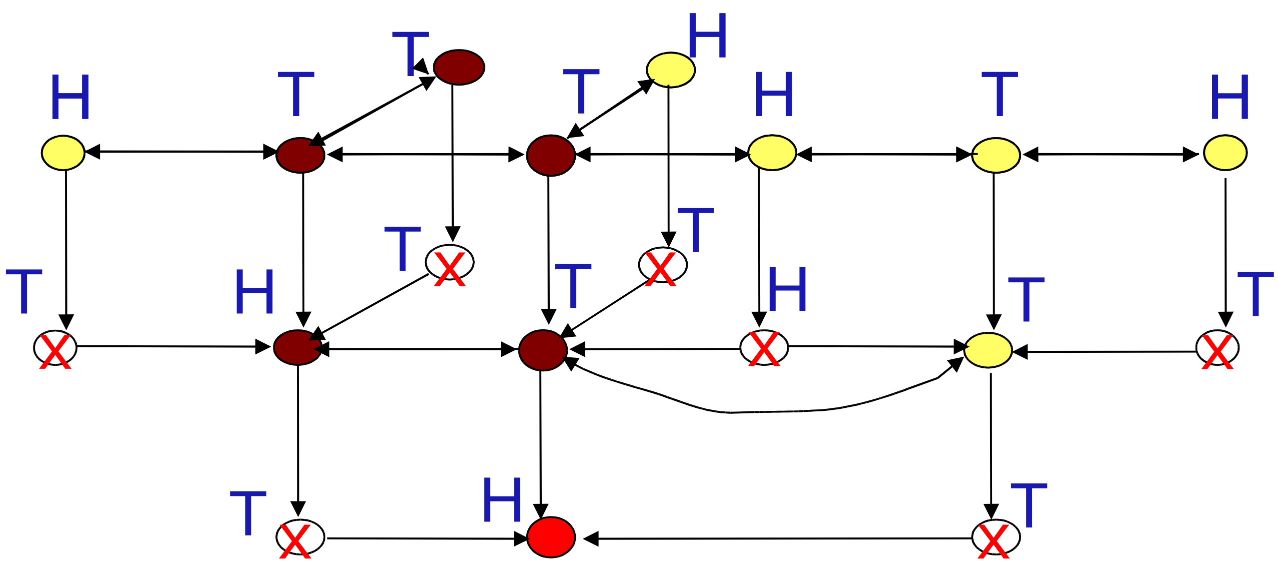
Three nodes woken up



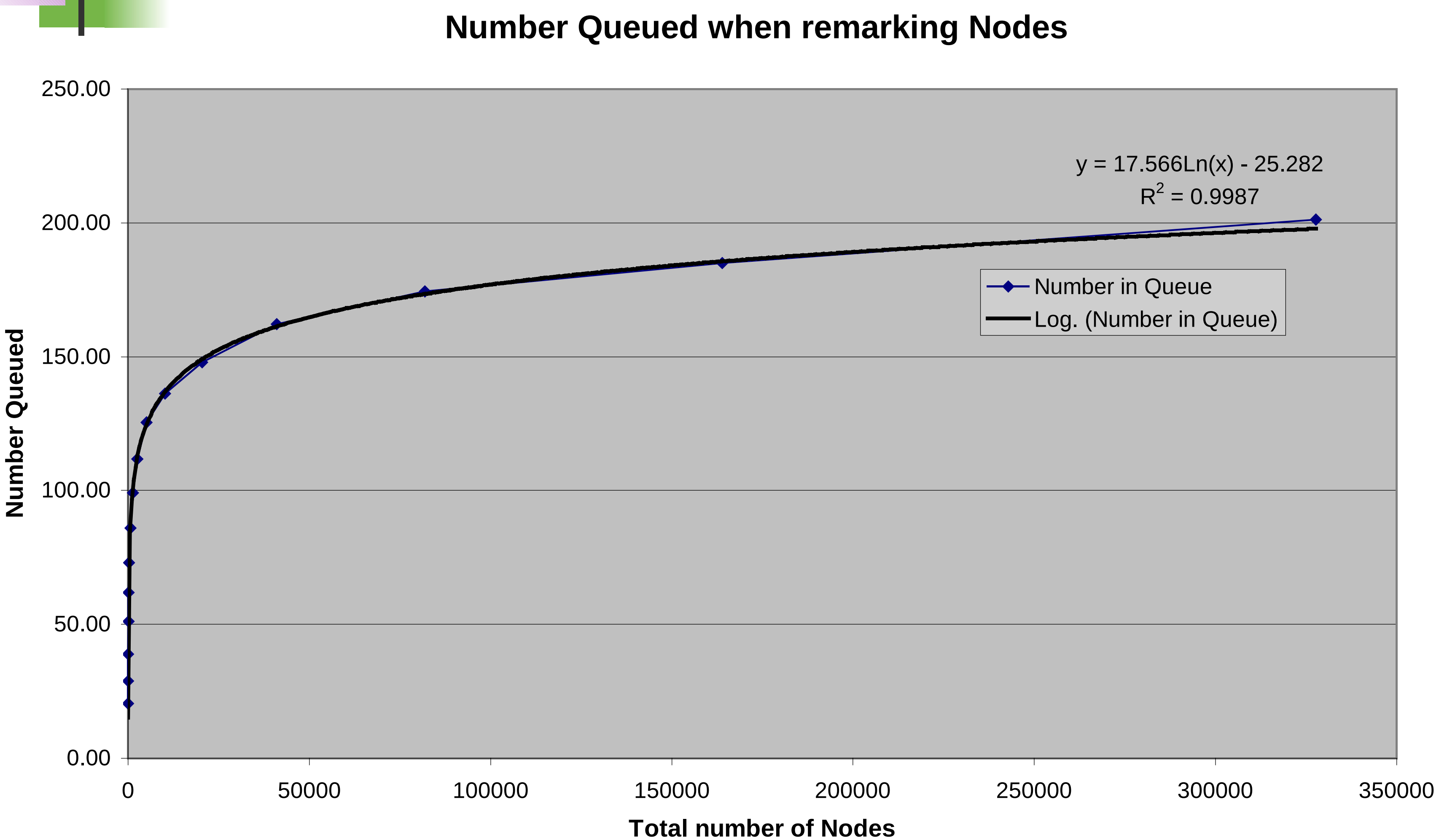
Nodes rerun code, more nodes woken up



Propagation continued



Experimental Results



Work In Progress

- Analyzing Power of the data structure (what it can and cannot do)
- Different Applications
- Analyzing Running times for:
  - Different changes.
  - Unbalanced Trees.

Conclusion and Future Work

- ConvexHull
  - Used adaptivity to solve the kinetic convex hull problem.
  - Encouraging results.
  - Adaptivity makes writing dynamic/kinetic algorithms a simple edition on the standard algorithm
  - The quickhull algorithm updates based on events efficiently in the expected case.
- **Parrallel Tree Contraction**
  - Efficient times ***O(log(n))*** expected time for an update.
  - Future Work:
    - More Applications