$OR \cap CS \cap ET =?$

December 1, 2003

There are three major goals of this meeting.

1. Facilitate the dissemination of recent results and techniques between the fields of economics and computer science.

2. See how these developments are aiding practitioners and whether we are answering questions relevant to practice.

3. Identify key open questions for research in the area of auctions and dynamic pricing mechanisms.

4. ....talk on the alternate viewpoints in approaching the research in auction coming from economists, computer scientists and operations researchers.

1. $OR =$ Operations Research
   Math of OR, **Math Programming**, **IPCO** and **Operations Research**.

2. $CS =$ Computer Science
   Mainly theoretical CS, i.e. **FOCS, STOC, SODA** and **COLT**. Notice conference proceedings.

3. $ET =$ Economic Theory
   Econometrica, JET, GEB and the like. Misses some areas like stochastic games, combinatorial games that have primarily mathematical motivations.

4. If it moves, optimize it.

5. If it moves, what does it optimize?

What is in the intersection of the three?

John von Neumann.

- Game Theory and Economic Behavior with Morgenstern.
- Duality theorem of linear programming.
- Cellular automata.

Stable Matchings and Roomates.

- Introduced by Gale and Shapley (1962) .

- Knuth's 1976 book: stable matching problem as a vehicle for analysis of algorithms.

- Gusfield and Irving (1989): algorithmic analysis of stable matchings and stable roomates.

- Van de Vate (1989) LP formulation of stable matching, Teo and Sethuraman (1998) LP formulation of stable roomates.

- Roth and Sotomayor (1990) book highlights strategic aspects of matching.

Learning.

- Blackwell Approachablity theorem (1956) Hannan (1958) theorem (known in CS as experts theorem), follows from Approachability.

- Fictitious Play (Robinson, 1958)

- Dantzig introduces interior point algorithm for LP that is a specialization of Approachability.

- Experts theorem in CS, Littleston and Warmuth (1989) = exponential fictitious play

- Calibration (Foster & Vohra (1992)), Checking rules (Kalai, Lehrer & Smorodinsky (1999)) Kolmogoroff complexity and randomness tests

- Good properties of Hannan, extensions to many experts (Kalai and Vempala (2003))

- Generalized Approchability (Lehrer (2002), Hart and Mas-Collel (2002)) connections to Interior Point methods

- PAC learning (Kalai (2003)), problem of integrability (Afriat's Theorem)

Bounded Rationality.

- Herbert Simon and satisficing.

- Bounded depth automata as models of rationality, Neyman (1985), Papadimitriou (1985).

- Dantzig-Wolfe decomposition, Papadimitriou, Yannakakkakis distributed computaion.

- van Zandt use of PRAM model

- Objections to complexity models:
  continuous input domain, unnatural to discretize the input in the problem definition
  worst-case
  what exactly does the number of states mean?
  computation $\neq$ cognition

- Like the weather, everyone talks about it but no one does anything about it.

Social Choice

- Early work using LP to characterize profiles on which majority rule does not cycle.

- Bartholdi, Trick and Tovey (1989), Hemaspaandra ^{2} (1997), Conitzer & Sandholm (2003) on complexity of voting algorithms,

- Redistricting problems and Integer programming from early 70's. Simeone & co-authors book on electoral procedures.

- Teo, Sethuraman & Vohra (2003) Integer and LP for domain characterizations.

$$ET \Rightarrow CS, OR$$

- New problems to solve/approximate: Core, Nucleolus, Nash

- New computational complexity questions: Classifiying non-dictatorial domains, compact representation of preferences

- Information as a constraint on optimization

  Take standard problem, add incentive constraints.

  Technically sweet, but is it relevant?

- Probabilistic Analysis of Equilibria
  Analogous to probabilitsic analysis of combinatorial problems
  Large bayesian games, equilibria may be hard to determine
  Say something about asymptotic properties of equilibrium outcomes

- Algorithm as model of dynamic process

  1. tatonement
  2. fictitious play
  3. proposal algorithm
  4. learning procedure
  5. top trading

6. coalition/network formation

7. iterative auctions

   single good, private values

   n bidders

   goal is efficiency, i.e., find largest number

   sealed bid second price auction is $O(n)$

   ascending english is $O(v_{max})$

   transcation costs ?

$$CS, OR \Rightarrow ET$$

- Complexity

  1. Computational Complexity

     know when 'local optimality' will not yield 'global optimality'

     design mechanisms that are 'hard' to manipulate

     a constraint on auction design

  2. Complexity of Description

     number of states needed to implement a strategy

     entropy of a strategy

3. Communication Complexity
   message length in mechanism design
   interactive proof systems

4. Trade-offs between computation and communication
   decentralizing computation
   Byzantine agreement, fault-tolerant communication

- Linear Programming

  1. Inequality descriptions of SWF's and SCF's

  2. Interpreting algorithms for LP's as iterative auctions

  3. Using network structures to characterize incentive compatibility

  4. Using network structures for multi-dimensiona screening

T is a set of (multi-dimensional) types.

T^n the set of all n-agent profiles of types.

Γ is a finite set of at least three outcomes.

An **allocation rule** is a function

$$f : T^n \to \Gamma.$$

For each $\alpha \in \Gamma$ there is a $\hat{t} \in T$ such that $f(\hat{t}) = \alpha$.

Payment rule

$$P : T^n \to \Re^n.$$

In profile $(t^1, \ldots t^n)$ agent i has type t^i she makes a payment of $P_i(t^1, \ldots, t^n)$.

Value that agent i with type $t \in T$ assigns to an allocation $\alpha \in \Gamma$ is $v^i(\alpha|t)$.

Dominant Strategy Incentive Compatible

For all agents i and all types $s \neq t$:

$$v^i(f(t,t^{-i})|t) - P_i(t,t^{-i}) \geq v^i(f(s,t^{-i})|t) - P_i(s,t^{-i}) \ \forall\ t$$

Suppress dependence on i, t^{-i}

$$v(f(t)|t) - P(t) \geq v(f(s)|t) - P(s)$$

$$v(f(t)|t) - P(t) \geq v(f(s)|t) - P(s) \quad (1)$$

$$v(f(s)|s) - P(s) \geq v(f(t)|s) - P(t). \quad (2)$$

Add (1) and (2)

$$v(f(t)|t) + v(f(s)|s) \geq v(f(s)|t) + v(f(t)|s).$$

$$v(f(t)|t) - v(f(s)|t) \geq -[v(f(s)|s) - v(f(t)|s)].$$

2-cycle inequality

$$v(f(t)|t) - v(f(s)|t) + [v(f(s)|s) - v(f(t)|s)] \geq 0.$$

f is dominant strategy IC if $\exists$ P's such that:

$$v(f(t)|t) - P(t) \geq v(f(s)|t) - P(s)$$

Fix f, find P's such that

$$P(t) - P(s) \leq v(f(t)|t) - v(f(s)|t) \ \ * .$$

A vertex for each type t

From vertex s to vertex t an edge of length $v(f(t)|t) - v(f(s)|t)$

From vertex t to vertex s an edge of length $v(f(s)|s) - v(f(t)|s)$

Glossing over infinities, system (*) is feasible iff the network just described has no (-)ve cycles.

2-cycle inequality

$$v(f(t)|t) - v(f(s)|t) + [v(f(s)|s) - v(f(t)|s)] \geq 0.$$

All 2-cycles in network are of non-negative length.

(Muller & Vohra 2003)

For many preference domains

2-cycles non (-)ve $\Rightarrow$ all cycles are non (-)ve

Lavi, Mu'alem and Nisan (2003)

Bikhchandani, Chatterji and Sen (2003)

QUESTIONS

1. 2-sided combinatorial markets
2. Relaxing the Common Prior Assumption
3. Information Acquisition
4. Collusion
5. Competition between Mechanisms
6. Withdrawal & Resale
7. Dynamics (markdowns, inventory, yield management)