APPROXIMATION ALGORITHMS FOR FACILITY LOCATION PROBLEMS

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APPROXIMATION ALGORITHMS & PERFORMANCE GUARANTEES

A ρ-APPROXIMATION ALGORITHM FOR AN OPTIMIZATION PROBLEM IS A POLYNOMIAL-TIME ALGORITHM THAT IS GUARANTEED TO FIND A FEASIBLE SOLUTION OF OBJECTIVE FUNCTION VALUE WITHIN A FACTOR OF ρ OF OPTIMAL.

THE PERFORMANCE GUARANTEE OF THE ALGORITHM IS ρ.

HEURISTIC – NEED NOT BE POLY-TIME (ALWAYS)
NEED NOT HAVE PERFORMANCE GUARANTEE

THREE ALGORITHMIC TECHNIQUES

- LP ROUNDING – ROUND FRACTIONAL OPT.
  TO "NEARBY" INTEGER SOL'N.

- PRIMAL-DUAL ALGORITHMS – USE LP
  AS ONLY IMPLICIT
  GUIDE TO FIND SOL'N

- LOCAL SEARCH PROCEDURES – ITERATIVELY IMPROVE
  SOL'N W.R.T.
  PRESET NOTION OF
  "NEARBY" SOLUTIONS

EUCLIDEAN CASE

[ARORA, RAGHAVAN, & RAO]

- QUASI-POLYNOMIAL APPROXIMATION SCHEME IN ANY CONSTANT DIMENSION

- POLYNOMIAL APPROXIMATION SCHEME IN $\mathbb{R}^2$

[KOLLIOPOULOS & RAO]

- POLYNOMIAL APPROXIMATION SCHEME IN CONSTANT DIMENSION

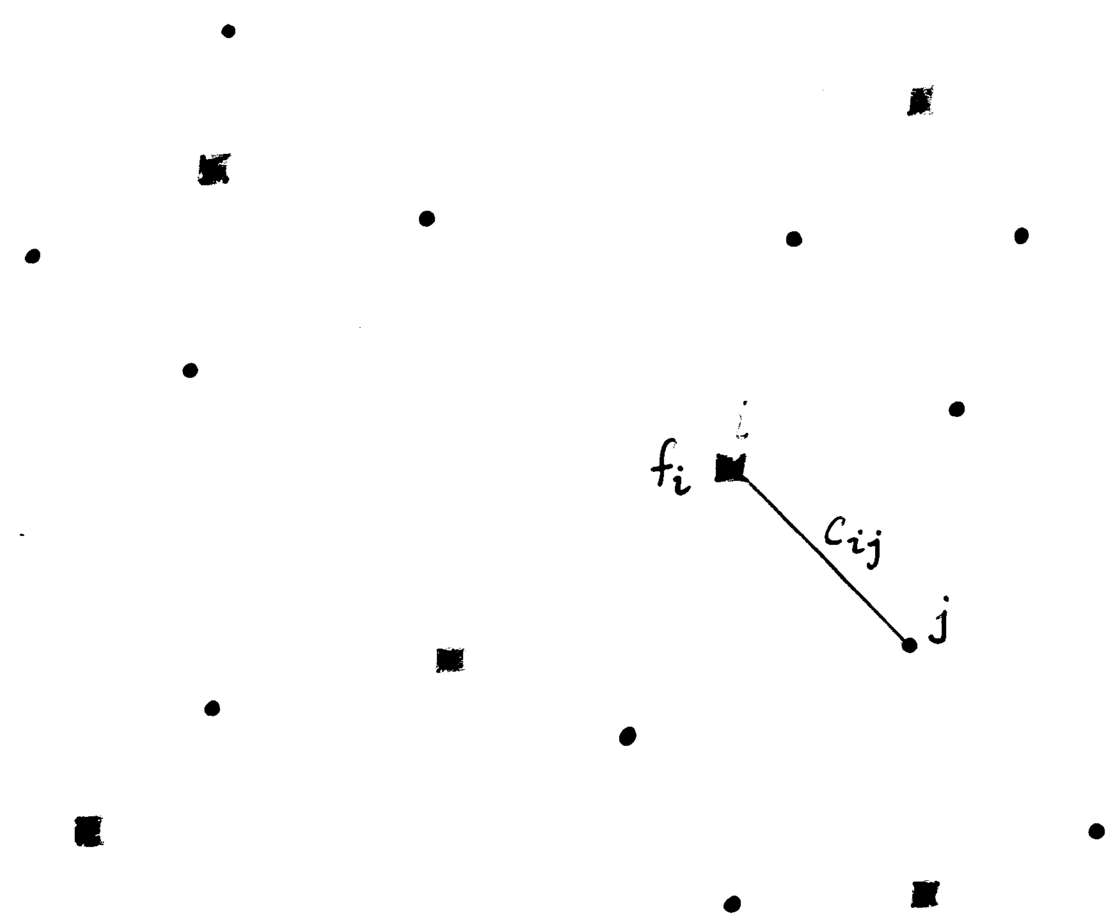
THE HIGHLIGHTS OF LP ROUNDING

- FORMULATE AS IP
- SOLVE LP RELAXATION
- ROUND LP OPTIMUM TO INTEGER SOL'N.
- PROVE ROUNDING DOES NOT INCREASE COST TOO MUCH
- LP PROVIDES LOWER BOUND (FOR MIN.)
- SO,

  BEING "CLOSE" TO LP VALUE

  $\Rightarrow$ BEING "CLOSE" TO OPTIMAL IP VALUE

THE UNCAPACITATED FACILITY LOCATION PROBLEM



KEY

- • CLIENTS TO BE ASSIGNED TO FACILITY
- ■ POTENTIAL SITES FOR FACILITIES
- f_i COST OF OPENING FACILITY AT i
- c_{ij} COST OF ASSIGNING CLIENT AT i TO FACILITY AT j

THIS PROBLEM HAS A LONG HISTORY

[BALINSKI, 1966]
[KUEHN & HAMBURGER, 1963]
[MANNE, 1964]
[STOLLSTEIMER, 1961]

EARLY FORMULATIONS & APPLICATIONS

[BALINSKI & WOLFE, 1963]
[EFROYMSON & RAY, 1966]
[SPIELBERG, 1969]
[REVELLE & SWAIN, 1970]
[MARSTEN, 1972]

EARLY COMPUTATIONAL WORK

- LAGRANGEAN RELAXATION
- PRIMAL-DUAL HEURISTICS
- PROBABILISTIC ANALYSIS OF ALGORITHMS
- COMPUTATIONAL COMPLEXITY (IT IS NP-HARD)
- PERFORMANCE GUARANTEES FOR APPROX. ALGORITHMS
- POLYHEDRAL STRUCTURE

AN INTEGER PROGRAMMING FORMULATION (BALINSKI, 1966)

MINIMIZE $$\sum_{i \in F} f_i y_i + \sum_{j \in D} \sum_{i \in F} c_{ij} x_{ij}$$

SUBJECT TO

$$\sum_{i \in F} x_{ij} = 1 \qquad \text{FOR EACH } j \in D$$

$$x_{ij} \leq y_i \qquad \text{FOR EACH } i \in F, j \in D$$

$$x_{ij}, y_i \in \{0, 1\} \quad \text{FOR EACH } i \in F, j \in D$$

DECISION VARIABLES

$$y_i = \begin{cases} 1 & \text{BUILD FACILITY AT SITE } i \\ 0 & \text{O.W.} \end{cases}$$

$$x_{ij} = \begin{cases} 1 & \text{ASSIGN CLIENT AT } j \text{ TO FACILITY AT } i \\ 0 & \text{O.W.} \end{cases}$$

DUAL LP

$$\text{MAXIMIZE} \quad \sum_{j \in D} v_j$$

$$\text{S.T.} \quad \sum_{j \in D} w_{ij} \leq f_i \qquad \text{FOR EACH } i \in F$$

$$v_j \leq c_{ij} + w_{ij} \qquad \text{FOR EACH } i \in F, j \in D$$

$$w_{ij} \geq 0 \qquad \text{FOR EACH } i \in F, j \in D$$

LEMMA IF $(\tilde{x}, \tilde{y})$ & $(\tilde{v}, \tilde{w})$ ARE OPTIMAL PRIMAL & DUAL LP SOLUTIONS, RESP.,

$$(*) \qquad \tilde{x}_{ij} > 0 \implies c_{ij} \leq \tilde{v}_j$$

PROOF FOLLOWS DIRECTLY FROM "COMPLEMENTARY SLACKNESS" OF LP OPTIMA

THE CLOSENESS PROPERTY

MOTIVATION FOR DUAL LP

EACH CLIENT j PROMISES TO PAY

"ITS SHARE"

OF FACILITY COST FOR i, f_i

EACH f_i REPLACED BY SHARES w_{ij}

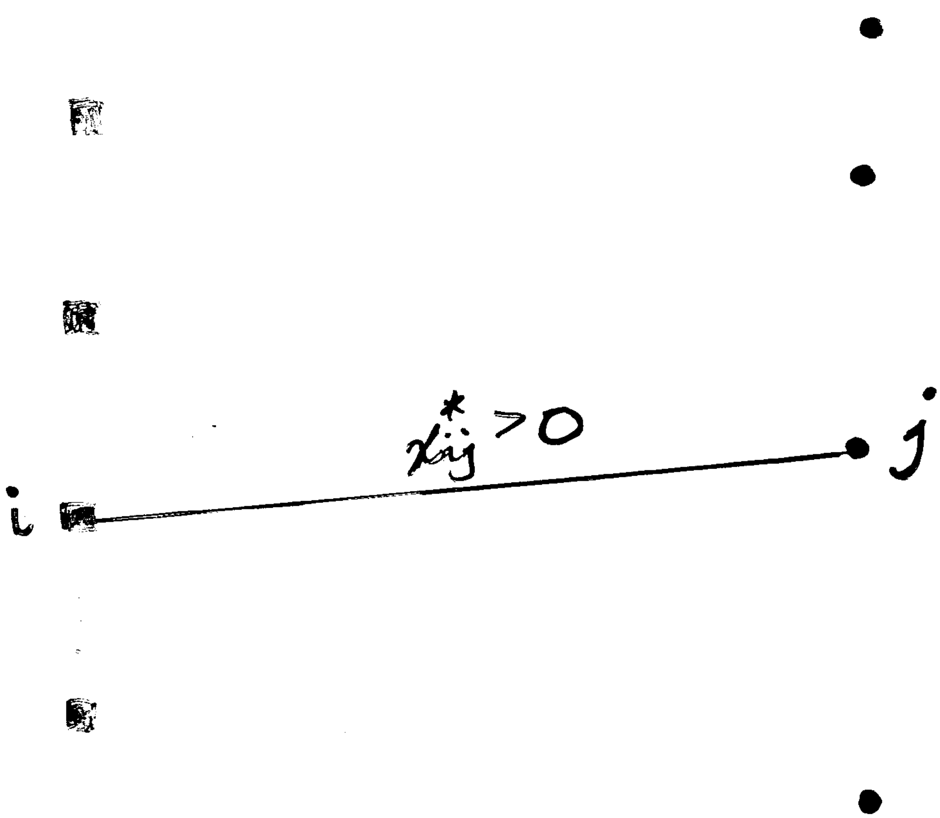
S.T. $\sum_j w_{ij} \leq f_i$

BUT WITHOUT FACILITY COSTS, EACH CLIENT j

USES FACILITY WITH MIN. ADJUSTED COST

$$\text{MIN}_i \{ c_{ij} + w_{ij} \}$$

VIEW FRACTIONAL SOLUTION AS A GRAPH

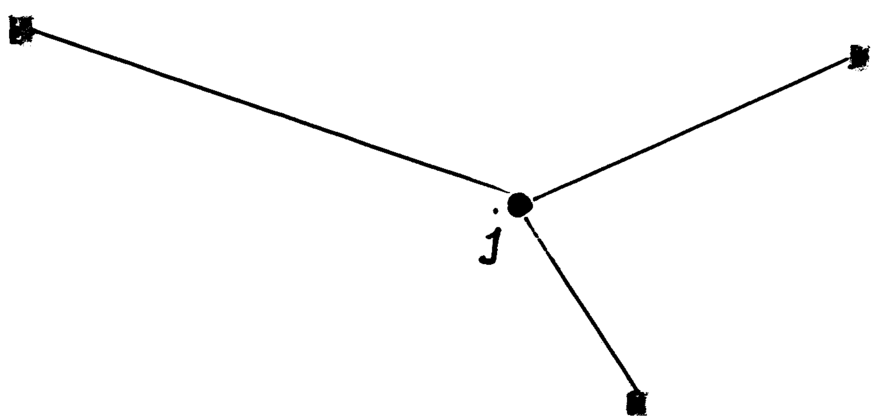


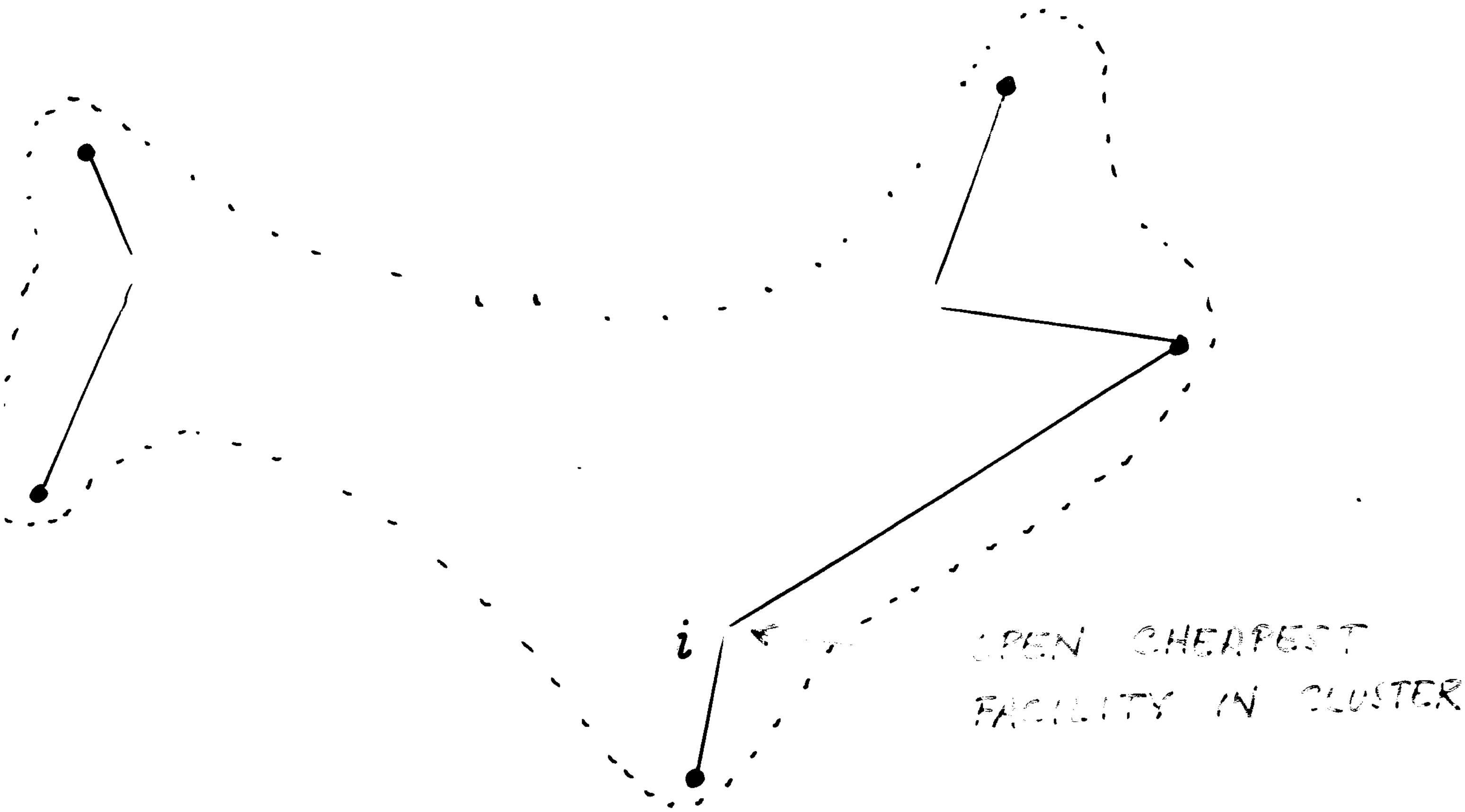
PUT EDGE (i,j) IN GRAPH WHENEVER $x^*_{ij} > 0$

ONE ITERATION

SELECT REMAINING DEMAND POINT j

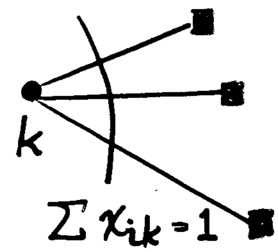
WITH MINIMUM $\tilde{v}_j$





INCLUDE IN CLUSTER

- NODE j
- ALL FACILITIES USED (FRACTIONALLY) BY j
- ALL DEMAND PTS. THAT USE THESE FACILITIES



INVARIANT: $\displaystyle\sum_{\substack{i:\ \text{REMAINING} \\ \text{NEIGHBOR OF } k}} \tilde{x}_{ik} = 1$ FOR ALL REMAINING k

k

$\leq \tilde{v}_k$

USE
CLOSENESS
PROPERTY

$\leq \tilde{v}_j$

$\leq \tilde{v}_j$

ASSIGNMENT COST OF $k \leq \tilde{v}_k + \tilde{v}_j + \tilde{v}_j \leq 3\,\tilde{v}_k$ BY CHOICE OF j

ℓ

$\tilde{x}_{\ell j} \leq \tilde{y}_\ell$

$$f_i = \mathrm{MIN}_{\ell:\ \mathrm{NEIGHBORS\ OF}\ j} \quad f_\ell \leq \sum_{\substack{\ell:\ \mathrm{N'BORS} \\ \mathrm{OF}\ j}} f_\ell\, \tilde{x}_{\ell j} \leq \sum_{\substack{\ell:\ \mathrm{N'BORS} \\ \mathrm{OF}\ j}} f_\ell\, \tilde{y}_\ell$$

TOTAL COST OF SOLUTION

$$\text{FACILITY COST} \leq \sum_{i \in F} f_i \tilde{y}_i \leq \text{LP-OPT}$$

$$\text{ASSIGNMENT COST} \leq \sum_{j \in D} 3 \tilde{v}_j \leq 3 \cdot \text{LP-OPT}$$

$$\text{TOTAL COST} \leq 4 \cdot \text{LP-OPT}$$

RANDOMIZED ROUNDING

FOR 0-1 INTEGER PROGRAM

- SOLVE FRACTIONAL (LP) RELAXATION
- INTERPRET FRACTIONAL VALUE $\tilde{y}_j$ AS PROBABILITY
- SET $y_j = 1$ WITH PROBABILITY $\tilde{y}_j$

[RAGHAVAN & THOMPSON, 1986]

SIMPLIFYING ASSUMPTION

A SOLUTION $(\tilde{x}, \tilde{y})$ IS COMPLETE

IF $\tilde{x}_{ij} > 0 \Rightarrow \tilde{x}_{ij} = \tilde{y}_i$

LEMMA FOR ANY INPUT I WITH LP OPTIMUM (x, y), THERE IS AN EQUIVALENT INPUT $\tilde{I}$ WITH COMPLETE LP OPTIMUM $(\tilde{x}, \tilde{y})$ (& THESE CAN BE EFFICIENTLY COMPUTED).

SO ...

W.L.O.G. MAY ASSUME INPUT HAS COMPLETE LP OPTIMUM

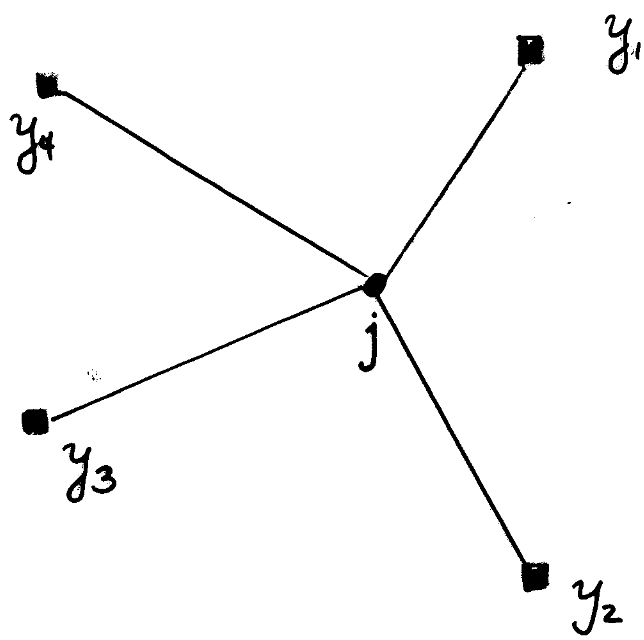
CLUSTERED RANDOMIZED ROUNDING

- FOR EACH CLUSTER, CONTAINING FACILITIES $S \subseteq F$, CHOOSE $i \in S$ WITH PROBABILITY $\tilde{y}_i$ & OPEN THERE (N.B. $\sum_{i \in S} \tilde{y}_i = 1$) (IND. FOR EACH CLUSTER)
- FOR EACH FACILITY i NOT IN <u>ANY</u> CLUSTER, OPEN AT i (INDEPENDENTLY) WITH PROB. $\tilde{y}_i$.
- ASSIGN EACH DEMAND POINT TO ITS NEAREST OPEN FACILITY

THEOREM. [CHUDAK & S]

$$E\left[\begin{array}{c}\text{TOTAL COST OF} \\ \text{SOLUTION FOUND}\end{array}\right] \leq \left(1+\frac{3}{e}\right) \text{LP-VALUE}$$

WHY IS CLUSTERED RANDOMIZED ROUNDING GOOD?



$$y_1 + y_2 + y_3 + y_4 = 1$$

$$\Rightarrow \quad \prod (1 - y_i) \leq 1/e$$

SO "MOST OF THE TIME" A 1-HOP ROUTING EXISTS,

AND WHEN THIS FAILS,

WE STILL HAVE 3-HOP ROUTING TO FALL BACK UPON!

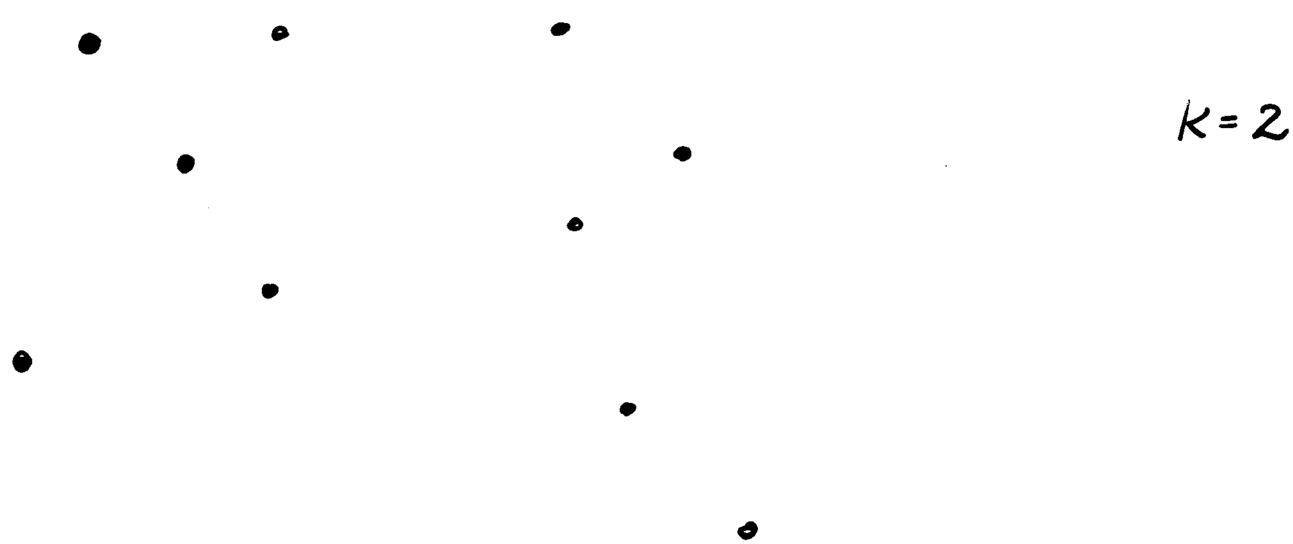
THE K-MEDIAN PROBLEM

GIVEN n POINTS IN A METRIC SPACE

CHOOSE k OF THEM (AS "MEDIANS")

& ASSIGN EACH POINT TO A MEDIAN

S.T. TOTAL ASSIGNMENT COST IS MINIMIZED



IP FORMULATION

MINIMIZE $$\sum_{i \in N} \sum_{j \in N} c_{ij} x_{ij}$$

S.T. $$\sum_{i \in N} x_{ij} = 1 \quad \text{FOR EACH } j \in N$$

$$x_{ij} \leq y_i \quad \text{FOR EACH } i, j \in N$$

$$\sum_{i \in N} y_i \leq k$$

$$x_{ij}, y_i \in \{0, 1\} \quad \text{FOR EACH } i, j \in N$$

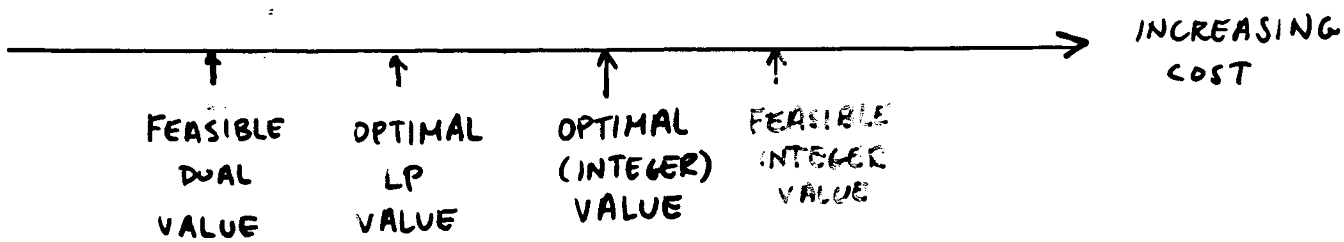
PRIMAL-DUAL ALGORITHMS

HIGH-LEVEL IDEA:

SIMULTANEOUSLY FIND

- FEASIBLE INTEGER SOLUTION TO ORIGINAL (PRIMAL) PROBLEM $\bullet$

- FEASIBLE SOLUTION TO DUAL LP $\bullet$



IF ALGORITHM IS GUARANTEED TO FIND $\bullet$ & $\bullet$

S.T. $\bullet \leq \rho \cdot \bullet \leq \rho \cdot \bullet$

$\Longrightarrow$ ALGORITHM IS ρ-APPROXIMATION ALGORITHM

[CHVATAL] [GOEMANS & WILLIAMSON]

PRIMAL

$$\text{MIN} \quad \sum_{i\in F} f_i y_i + \sum_{j\in D} \sum_{i\in F} c_{ij} x_{ij}$$

SUBJECT TO

$$\sum_{i\in F} x_{ij} = 1 \qquad \text{FOR EACH } j\in D$$

$$0 \le x_{ij} \le y_i \qquad \text{FOR EACH } i\in F, j\in D$$

DUAL

$$\text{MAX} \quad \sum_{j\in D} v_j$$

S.T.

$$\sum_{j\in D} w_{ij} \le f_i \qquad \text{FOR EACH } i\in F$$

$$v_j \le c_{ij} + w_{ij} \qquad \text{FOR EACH } i\in F, j\in D$$

$$w_{ij} \ge 0 \qquad \text{FOR EACH } i\in F, j\in D$$

A PRIMAL-DUAL ALGORITHM FOR THE UNCAPACITATED FACILITY LOCATION PROBLEM
[JAIN & VAZIRANI]

THM. ALGORITHM FINDS

- INTEGER (x_{ij}, y_i) &
- ARBITRARY (v_j, w_{ij})

S.T. BOTH ARE FEASIBLE &

$$\sum_{ij} c_{ij} x_{ij} + 3 \sum_i f_i y_i \leq 3 \sum_j v_j$$

COROLLARY PRIMAL-DUAL ALGORITHM HAS PERFORMANCE GUARANTEE OF 3.

WHY ARE k-MEDIAN & UNCAP. FACILITY PROBLEMS RELATED?

A SIMPLE, BUT POWERFUL, IDEA [JAIN & VAZIRANI]

$$f_i,\ c_{ij} \longrightarrow \boxed{\begin{array}{l}\text{ALGORITHM FOR}\\ \text{UNCAPACITATED}\\ \text{FACILITY LOCATION PROBLEM}\end{array}}$$

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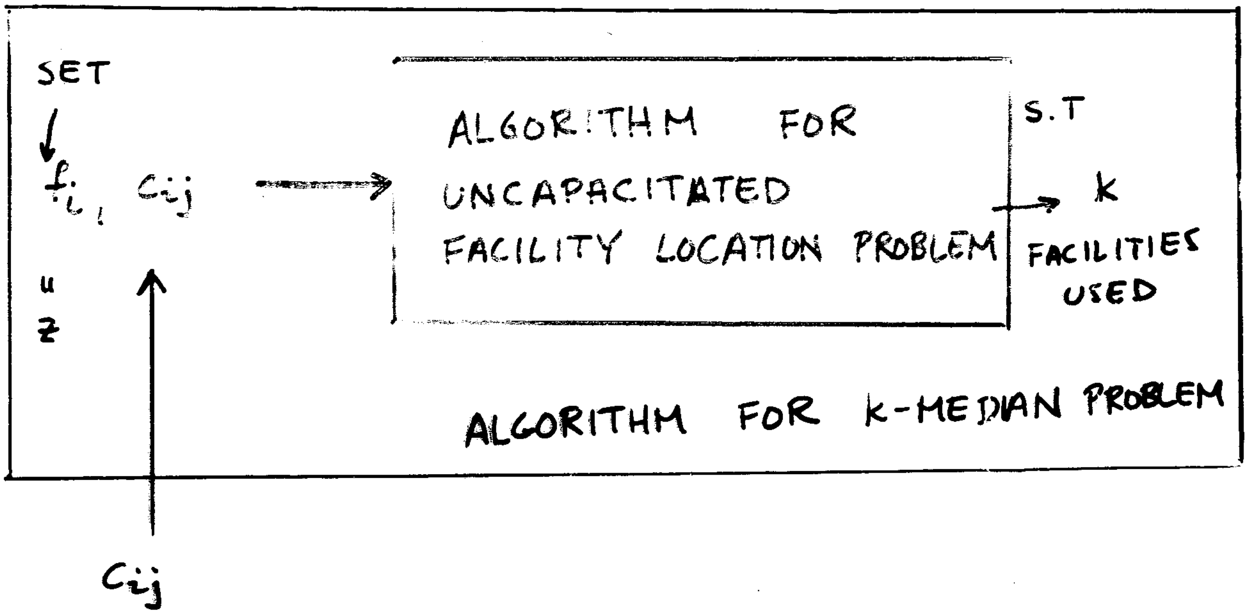


- DESIGN "PRIMAL-DUAL" SUBROUTINE
- ANALYSIS FURTHER LINKS TWO PROBLEMS

A TALE OF 4 LPs

UNCAP. FAC. LOC.

$$\text{MIN} \quad \sum_{i,j} c_{ij} x_{ij} + \sum_i f_i y_i$$

$$\text{S.T.} \quad \sum_i x_{ij} = 1 \qquad \forall j$$

$$0 \le x_{ij} \le y_i \qquad \forall i,j$$

K-MEDIAN

$$\text{MIN} \quad \sum_{i,j} c_{ij} x_{ij}$$

$$\text{S.T.} \quad \sum_i x_{ij} = 1 \qquad \forall j$$

$$0 \le x_{ij} \le y_i \qquad \forall i,j$$

$$\sum_i y_i \le k$$

DUAL

$$\text{MAX} \quad \sum_j v_j$$

$$\text{S.T.} \quad v_j - w_{ij} \le c_{ij} \qquad \forall i,j$$

$$\sum_j w_{ij} \le f_i \qquad \forall i$$

$$v_j, w_{ij} \ge 0 \qquad \forall i,j$$

DUAL

$$\text{MAX} \quad \sum_j v_j - zk$$

$$\text{S.T.} \quad v_j - w_{ij} \le c_{ij}$$

$$\sum_j w_{ij} \le z$$

$$v_j, w_{ij}, z \ge 0 \qquad \forall i,j$$

WHY DOES THIS YIELD GUARANTEE FOR k-MEDIAN ?

TAKE k-MEDIAN INPUT & SET $f_i = z$ S.T.

PRIMAL-DUAL ALGORITHM USES EXACTLY k FACILITIES

(OUTPUT IS (x,y) & (v,w))

$\Rightarrow$ (x,y) IS FEASIBLE INTEGER SOLUTION TO k-MEDIAN

(v,w,z) IS FEASIBLE SOL'N FOR k-MEDIAN DUAL

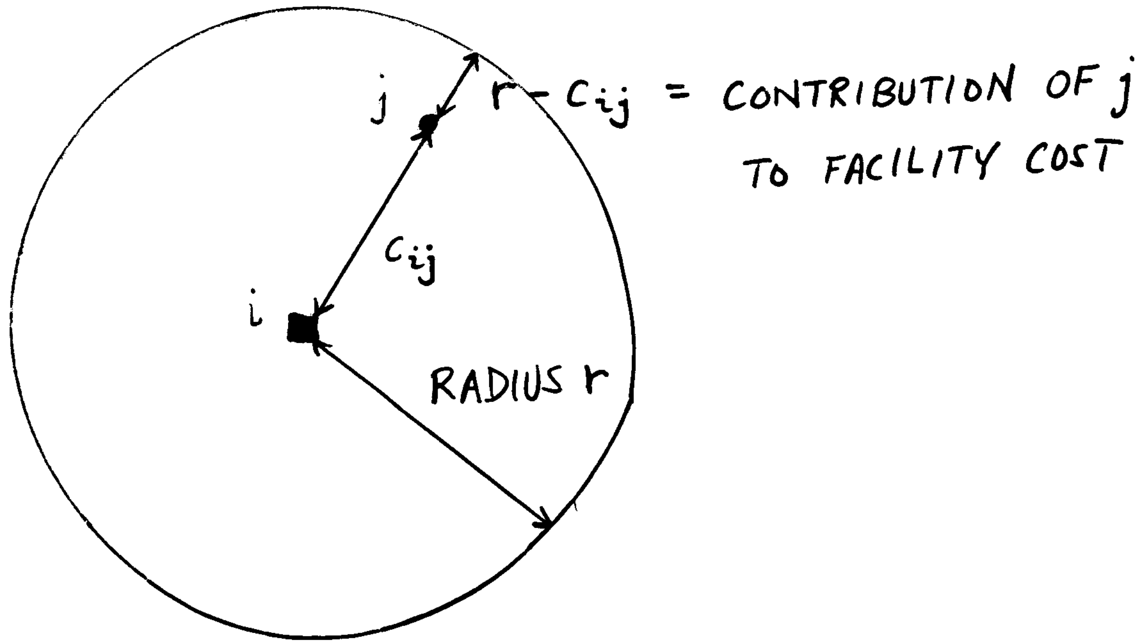
BY THM

$$\sum_{i,j} c_{ij} x_{ij} + 3\sum_i z y_i \le 3 \sum_j v_j$$

$\Rightarrow$

$$\sum_{i,j} c_{ij} x_{ij} \le 3\Big(\sum_j v_j - z\underbrace{\sum_i y_i}_{=k} \Big) = 3\Big(\sum_j v_j - zk \Big)$$

RADIUS OF FACILITY



FOR EACH FACILITY i

SET RADIUS $r_i \leftarrow$ RADIUS r S.T. TOTAL CONTRIBUTIONS OF CLIENTS $= f_i$

$B_i \leftarrow$ "BALL" OF RADIUS r_i

WLOG ASSUME $r_1 \leq r_2 \leq \cdots \leq r_m$

ALGORITHM OF METTU - PLAXTON

SET $S \leftarrow \emptyset$

ITERATE THROUGH ALL $i = 1, \ldots, m$

IF $2B_i \cap S = \emptyset$ THEN

ADD i TO S

AN IMPLICIT DUAL SOLUTION

[ARCHER, RAJAGOPALAN, & S]

$$v_j = \min_i \{ c_{ij} + w_{ij} \}$$

WHERE

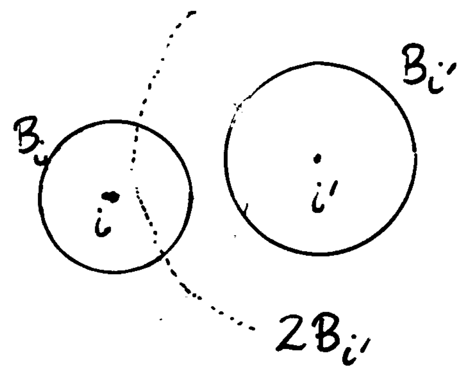
$$w_{ij} = \begin{cases} r_i - c_{ij} & \text{IF } j \in B_i \\ 0 & \text{O.W.} \end{cases}$$

LET $i(j)$ DENOTE i THAT ACHIEVES MIN

(IF TIES, TAKE LOWEST INDEX)

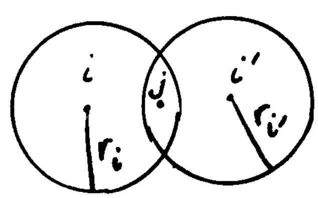
LEMMA IF i & i' ARE BOTH OPEN FACILITIES THEN $c_{ii'} \geq \max\{r_i, r_{i'}\}$

PROOF SUPPOSE i IS OPENED FIRST.



COROLLARY EACH DEMAND POINT IS IN B_i FOR AT MOST ONE OPEN FACILITY i

PROOF SUPPOSE NOT



3 TYPES OF DEMAND POINTS j

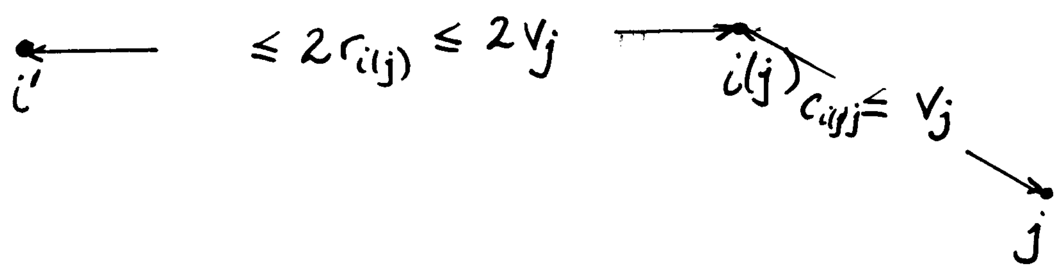
CASE 1: $i(j)$ IS OPEN

- ASSIGN j TO $i(j)$
- $v_j = c_{i(j),j} + w_{i(j),j}$

LEMMA LET j BE S.T. $i(j)$ NOT OPEN & SO $\exists\, i'$ THAT IS OPEN THAT "CLOSED" $i(j)$. THEN

$$c_{i'j} \leq 3 v_j$$

PROOF $v_j = c_{i(j),j} + w_{i(j),j} = \text{MAX} \{ r_{i(j)} , c_{i(j),j} \}$



CASE 2: $i(j)$ IS CLOSED & ($j \in int(B_i) \Rightarrow i$ CLOSED)

SUPPOSE THAT OPEN FACILITY i "CLOSES" $i(j)$

- ASSIGN j TO i
- $c_{ij} \le 3 v_j$

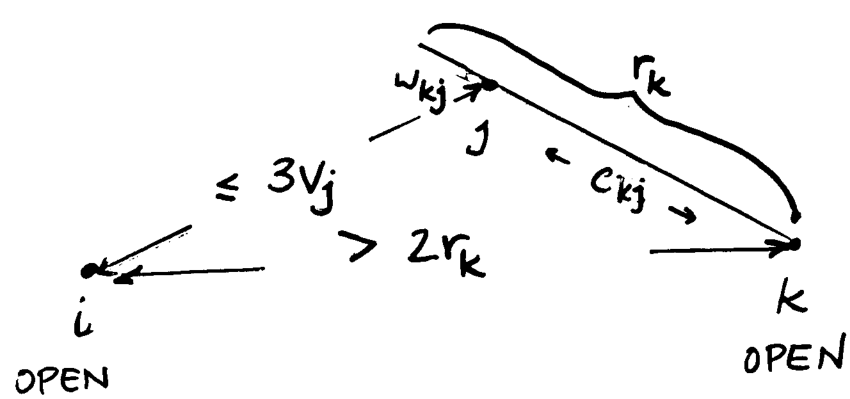
CASE 3: $i(j)$ IS CLOSED & $j \in int(B_k)$ s.t. k OPEN

SUPPOSE THAT OPEN FACILITY i "CLOSES" $i(j)$

LEMMA $r_i < r_k$ (& HENCE $i \ne k$)

PROOF $r_i \le r_{i(j)} \le v_j < r_k$

BY DEF'N OF $i(j)$ & $j \in int(B_k)$



$$2r_k < c_{ik} \leq c_{kj} + 3v_j = (r_k - w_{kj}) + 3v_j$$

$$\Rightarrow \quad r_k + w_{kj} < 3v_j$$

$$\parallel$$

$$c_{kj} + 2w_{kj}$$

- ASSIGN j TO k
- $c_{kj} + 2w_{kj} < 3v_j$

CLAIM $\sum_j 3v_j \geq \sum_j \Big(\text{ASSIGNMENT COST FOR } j + 2 \sum_{i:\text{OPEN}} w_{ij}\Big)$

$$= \sum_{i,j} c_{ij}\bar{x}_{ij} + 2\sum_i f_i \bar{y}_i$$

[PAL & TARDOS] INDEPENDENTLY USED SAME DUAL TO GIVE "STRATEGY-PROOF" MECHANISM

LOCAL SEARCH FOR k-MEDIAN PROBLEM
[ARYA, GARG, KHANDEKAR, PANDIT, MEYERSON, & MUNAGALA]

SIMPLE SWAP MOVE – DELETE ONE CENTER
ADD ANOTHER ONE

"RECALL" NOTATION

S – CENTERS IN CURRENT SOLUTION

S^* – CENTERS IN FIXED OPTIMAL SOLUTION

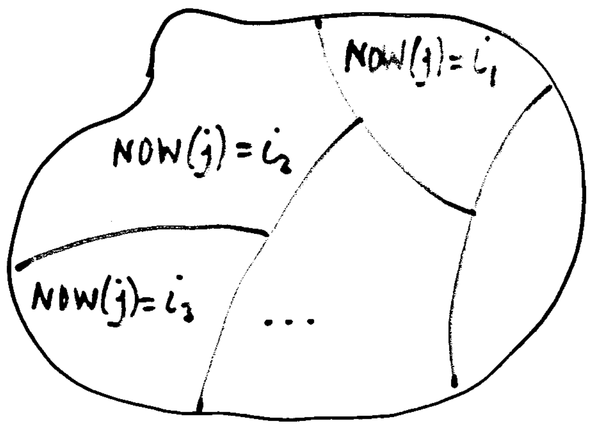
C – COST OF CURRENT SOLUTION

C^* – COST OF OPTIMAL SOLUTION

NOW(j) – CENTER SERVING j IN CURRENT SOL'N

OPT(j) – CENTER SERVING j IN OPTIMAL SOL'N

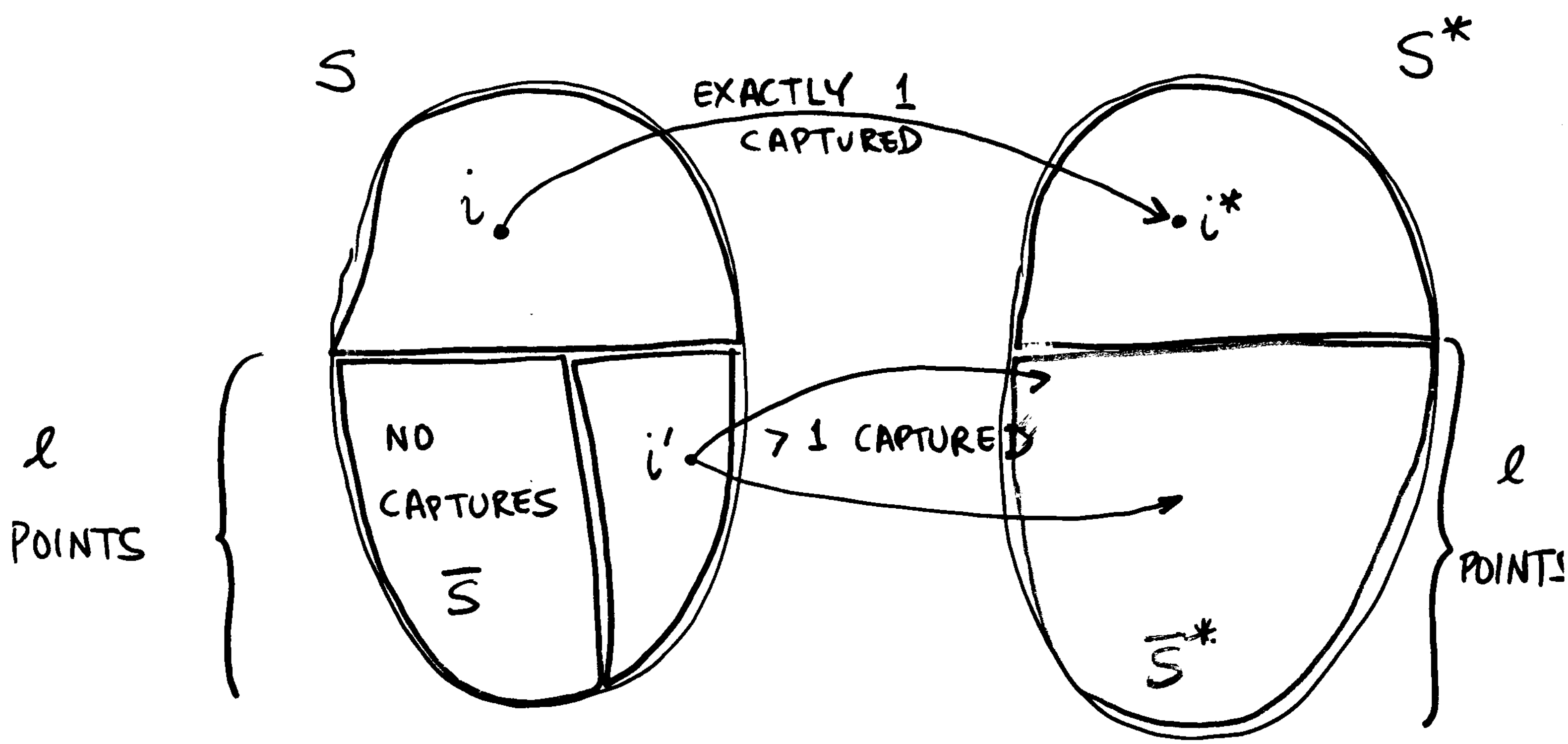
PARTITION OF POINTS SERVED BY i^* IN OPTIMAL SOL'N



$j : OPT(j) = i^*$

SIMPLE PARTITION – SIZE OF EACH PART $\le \frac{1}{2}$ TOTAL SIZE

IF SIZE OF PART CORRESPONDING TO i $> \frac{1}{2}$ TOTAL SIZE $\Longrightarrow$ i CAPTURES i^*



CLAIM $|\bar{S}| \geq \ell/2$ (BY EASY COUNTING ARG.)

COROLLARY CAN "PAIR UP" EACH POINT IN $\bar{S}$

WITH 1 OR 2 POINTS IN $\bar{S}^*$ S.T.

EACH PT. IN $\bar{S}^*$ IS PAIRED EXACTLY ONCE.

OUR k <u>CRUCIAL</u> SWAPS

- i WITH i^* WHEN i CAPTURES i^* & NOTHING ELSE

- i WITH i^* WHERE i CAPTURES NOTHING

NOTE : EACH i^* IS IN EXACTLY ONE SWAP

EACH i IS IN ≤ 2 SWAPS

MAGIC PERMUTATION

π : MAPS EACH $\qquad j \to j'$

j (IN NOT BIG PART)
TO j' IN ANOTHER PART

SWAP i & i^*

- ALL j S.T. $OPT(j) = i^*$ GET ASSIGNED TO i^*
- FOR EACH j S.T. $NOW(j) = i$ & $OPT(j) = i'$ $(\neq i^*)$

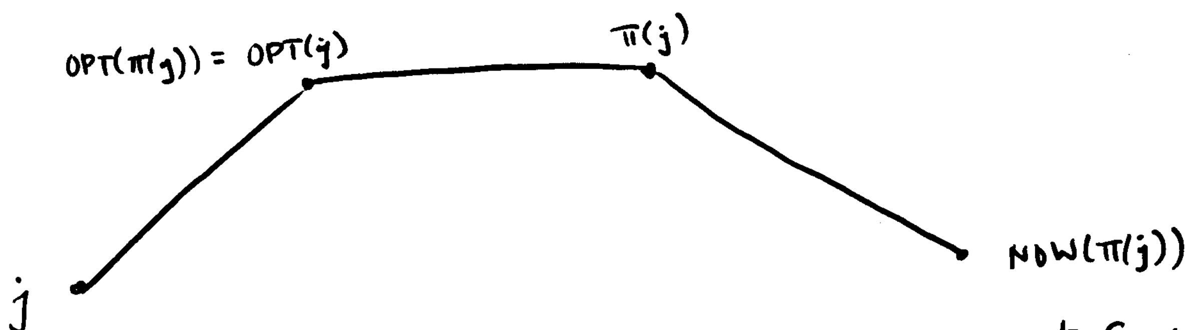
  NOW ASSIGN j TO $NOW(\pi(j))$

CLAIM IN LATTER CASE, $NOW(\pi(j)) \neq i$.

PROOF $NOW(j)$ DOES NOT CAPTURE i'

HENCE MAGIC PERMUTATION π MAPS j TO $\pi(j)$

WITH $NOW(\pi(j)) \neq NOW(j) = i$



$$\Rightarrow \quad c_{j\,NOW(\pi(j))} \leq c_{j\,OPT(j)} + c_{\pi(j)\,OPT(\pi(j))} + c_{\pi(j)\,NOW(\pi(j))}$$

$$\rotatebox{-90}{$\geq$}$$

$$c_{j\,NOW(j)}$$

CHANGE OF COST FOR SWAP OF i & i^*

$$\sum_{\substack{j:OPT(j)\\=i^*}} \left(c_{jOPT(j)} - c_{jNOW(j)}\right) + \sum_{\substack{j:NOW(j)=i\\ \& \, OPT(j)\neq i^*}} \left(c_{jNOW(\pi(j))} - c_{jNOW(j)}\right) \geq 0$$

$$\wedge\!|$$

$$\sum_{\substack{\text{CRUCIAL}\\ \text{SWAPS}}} \left(\sum_{\substack{j:OPT(j)\\=i^*}} \left(\underbrace{c_{jOPT(j)}}_{C^*} - \underbrace{c_{jNOW(j)}}_{C}\right) + \sum_{\substack{j:NOW(j)=i\\ \& \, OPT(j)\neq i^*}} \underbrace{c_{jOPT(j)}}_{2C^*} + \underbrace{c_{\pi(j)OPT(\pi(j))}}_{2C^*} + \underbrace{c_{\pi(j)NOW(\pi(j))} - c_{jNOW(j)}}_{0} \right) \geq 0$$

$$\Rightarrow \quad 5C^* - C \geq 0$$

$$C \leq 5C^*$$

GENERALIZATIONS

- MULTI-LEVEL FACILITY LOCATION

  [AARDAL, CHUDAK, & S]

  [MEYERSON, MUNAGALA, & PLOTKIN]

  [GUHA, MEYERSON, & MUNAGALA]

  [BUMB & KERN]

- CAPACITATED FACILITY LOCATION

  "SOFT" VS. "HARD" CAPACITIES

"SOFT" [S, TARDOS, & AARDAL]

[CHUDAK & S]

[JAIN & VAZIRANI]*

[KORUPOLU, RAJARAMAN, & PLAXTON]

[CHUDAK & WILLIAMSON]

"HARD" [PAL, TARDOS, & WEXLER]

- FAULT-TOLERANT FACILITY LOCATION

  [JAIN & VAZIRANI]

  [GUHA, MEYERSON, & MUNAGALA]

  [SWAMY & S]

  [JAIN, MAHDIAN, MARKAKIS, SABERI, & VAZIRANI]

- FACILITY LOCATION WITH PENALTIES (& k-MEDIAN WITH OUTLIERS)

  [CHARIKAR, KHULLER, MOUNT, NARASIMHAM]

  [JAIN, MAHDIAN, MARKAKIS, SABERI, & VAZIRANI]

- DATA PLACEMENT

- MINIMIZE SUM OF CLUSTER DIAMETERS

⋮

OPEN PROBLEMS

- FIND APPROXIMABILITY THRESHOLD FOR FACILITY LOCATION & k-MEDIAN PROBLEMS

- "LP-BASED" GUARANTEE FOR "HARD" CAPACITATED FACILITY LOCATION

- EXPLOIT "ODD-CYCLE" CONSTRAINTS FOR UNCAPACITATED PROBLEMS

- ANALYZE LAGRANGEAN RELAXATION ? FOR CAPACITATED FACILITY LOCATION

- CAN RANDOMIZATION PLAY A ROLE IN PRIMAL-DUAL ALGORITHMS ? LOCAL SEARCH ALGORITHMS ?

OVERVIEW

	UFL		k-MEDIAN	
LP ROUNDING	S, TARDOS, & AARDAL	4	CHARIKAR, GUHA, TARDOS & S	$6\frac{2}{3}$
		3.16		
	GUHA & KHULLER	2.46^*		
	CHUDAK & S	$1+2/e$		
		$1+2/e-.0001^*$		
	SVIRIDENKO	$1.67 \rightsquigarrow 1.582$		
LOCAL SEARCH	KORUPOLU, PLAXTON & RAJARAMAN	5	ARYA, GARG, KHANDEKAR, PANDIT, MEYERSON & MUNAGALA	3
	CHARIKAR & GUHA	3		
		$1+\sqrt{2}^{\,+}$		
PRIMAL-DUAL	JAIN & VAZIRANI	3	JAIN & VAZIRANI	6
	METTU & PLAXTON	3	CHARIKAR & GUHA	4
	MAHDIAN, MARKAKIS, SABERI, & VAZIRANI	1.861	JAIN, MAHDIAN, & SABERI	4^+
	JAIN, MAHDIAN, & SABERI	1.61		
"KITCHEN SINK"	MAHDIAN, YE, ZHANG	1.52		
LOWER BOUNDS	GUHA & KHULLER	1.463	JAIN, MAHDIAN, & SABERI	$1+2/e$

* ADD "GREEDY IMPROVEMENT" POST-PROCESSING

\+ ADD RESCALING PRE-PROCESSING