Network Design Models for Distribution Systems

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Outline

- Motivation
- Problem definition
- Network Design model
- Variants and special cases

- Overview of solution methods
- Dual Ascent approach
- Computational results
- Importance of good model formulation
- Extensions

Based on joint work with Thomas Magnanti, Prakash Mirchandani, Richard Wong, and others

Distribution System Design example
Armstrong World Industries

- Market leader in flooring and ceiling products
- > $ 3 Billion sales in 2001
- Product-focused organization structure
  - **Flooring** products division versus **Ceiling** division
  - distinct manufacturing, sales, distribution organizations
- Traditional distribution system
  - independent flooring and ceiling distributors
  - responsible for sales, pricing, delivery, credit
- Big-Box retailers are now biggest customers

Big-Box Retailers

- Growing rapidly, increasing market power
  - **Home Depot**: $54 billion annual sales, 1400+ US stores and growing
- Formula for success: *wide variety at competitive prices under one roof*
  - negotiate low prices from manufacturers
  - maintain low store inventory, high variety
  - ensure high availability
- "Demanding" customers
  - need a single point of contact
  - *frequent* (weekly) deliveries of *multiple* items in *small* batches
  - 24-hour delivery lead time

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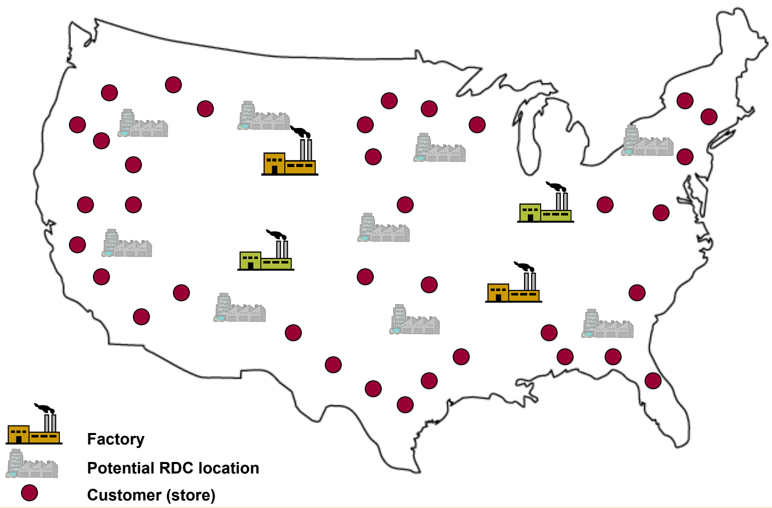
Meeting Big-Box Retailers' needs

- Set up **new** distribution network
  - Establish **Regional Distribution Centers** (**RDC**) for warehousing, distribution
  - Specify coverage region (assign stores) for each RDC
  - RDCs receive bulk shipments from factories
  - RDCs deliver orders to stores (small trucks) at scheduled times (weekly)
- + Organizational changes

Distribution System Design issues

- **Decisions**
  - How many distribution centers?
  - Where?
  - Which customers (stores) to assign to each DC?
  - Which plants to supply each DC?
- **Problem scope**
  - 19 Flooring plants, 7 Ceiling plants in U.S. alone
  - Hundreds of SKUs
  - Over 2000+ customer locations (stores) nationwide

Distribution System Design problem



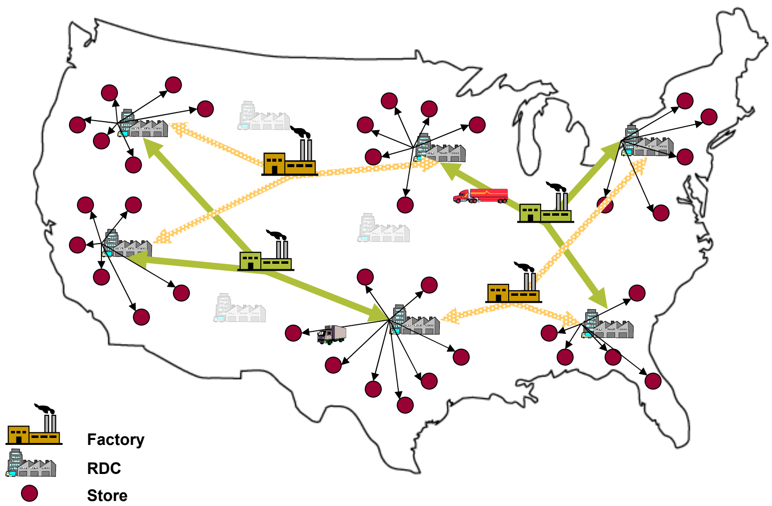
Cost tradeoffs

- **Cost components**
  - **DC costs: Fixed** investment + operating costs (fixed and/or throughput-dependent)
  - **Inbound** (plant-to-DC) **transportation** costs: including economies of scale
  - **Outbound** (DC-to-store) **delivery** cost

- **Basic tradeoff**

  If we open more DCs ➔ we can locate them closer to customers, but …
  - Higher total DC fixed costs
  - Higher total inbound transportation costs
  - Lower outbound delivery cost

Distribution System Design solution



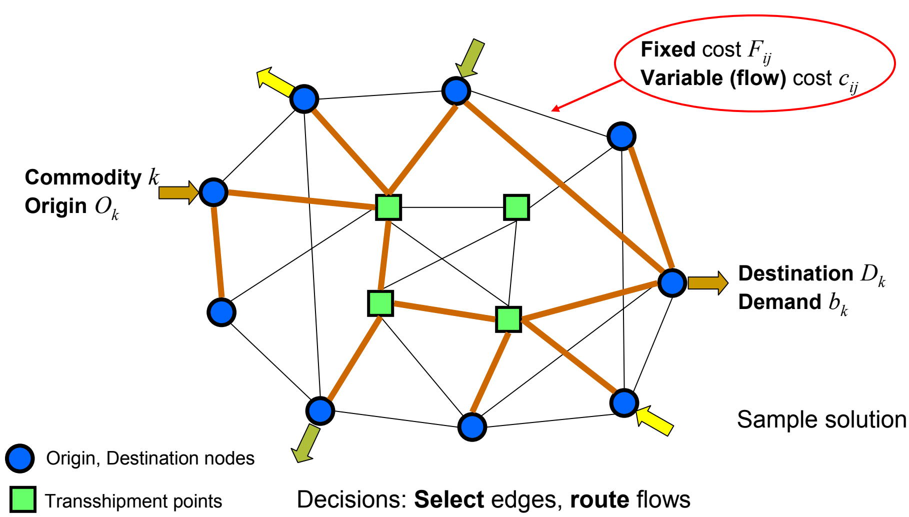
Strategic Distribution System Design applications

- Hunt-Wesson Foods – Geoffrion and Graves (1974)
- Yellow Freight – Powell et al. (1992)
- Digital Equipment Corp. - Brown et al. (1995)
- Procter & Gamble – Camm et al. (1997)
- UPS – Barnhart et al. (1999, 2002)
- Railroad Blocking – Barnhart et al. (2000)
- DHL Hong Kong – Cheung et al. (2001)

Practice of Supply Chain Design

- Models vary depending on application context
- Large problem sizes ('00s of nodes, '000s of O-D pairs)
- Need specialized solution methodologies to exploit problem structure
- Reported savings of tens to hundreds of millions of dollars
- Optimal design capability supplemented with:
  - Detailed simulations to test optimal designs
  - What-if, sensitivity, and scenario analyses
- Commercial SC and ERP systems now provide APS modules with SC design optimization capability

Network Design Problem definition



Decisions: **Select** edges, **route** flows

Network Design Problem definition

- Given
  - NODES
    - Origins (plants), transshipment points (DC), destinations (stores)
  - COMMODITIES
    - *Origin* and *destination*
    - Demand
  - EDGES
    - *Fixed* cost
    - *Variable* (flow) cost
- Required
  - *Select the edges* to use
  - *Route* required flows
- Objective
  - Minimize total FIXED cost of design + FLOW costs

Notation

Parameters

- $G:(N,E)$ Given graph (directed or undirected)
- $i,j \in N$ Nodes
- $(i,j) \in N$ Edges (directed) or arcs (directed)
- A Node-arc incident matrix
- $k \in K$ Commodities
- B_k Demand vector for commodity k
- F_{ij} Fixed cost of edge (i,j)
- c_{ij}^k Variable cost of comm. k from i to j on edge (i,j)

Variables

- f_{ij}^k Units of f**low** of comm. k from i to j on edge (i,j)
- z_{ij} **Design** variable;
  = 1 if solution includes edge (i,j), 0 otherwise

Arc flow formulation

$$\text{Minimize} \quad Fz + \sum_{k \in K} c^k f^k$$

Flow conservation constraints	$Af^k = B_k$	$\forall k \in K$
Forcing constraints	$f^k \leq u^k z$	$\forall k \in K$
Nonnegativity, integrality	$f^k \geq \mathbf{0} \ \forall k \in K, z \in \{0,1\}^m$	
Routing requirements (optional)	$f^k \in P^k$	$\forall k \in K$
Design restrictions (optional)	$z \in Z(\mathbf{f})$	

Forcing coefficients

Permissible flow paths

Permissible topologies

Classification of Network Design problems

- **Demand** structure
  - Complete demand, single source, single destination, ..
  - With or without Steiner nodes
- **Network** structure
  - Directed vs. undirected
  - Tree, layered, general
  - With or without edge duplication
  - Multiple node types, facility types
- **Cost** structure
  - Only fixed costs, fixed + variable costs, step function, general concave costs
- **Design** and **Routing** restrictions

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Types of additional constraints

- **Routing requirements**
  - Weight (e.g., delay) or hop constraints
  - Bifurcated vs. non-bifurcated flows
  - Alternate (edge-disjoint) paths
  - Commodity-dependent facility requirements
- **Design restrictions**
  - Capacity constraints—commodity-specific, bundled
  - Simultaneous vs. non-simultaneous usage
  - Degree, diameter constraints
  - Precedence, multiple choice constraints
  - Topological restrictions: tree, ring, ...

Other application contexts for Network Design models

- Supply chain design
- Less-than-truckload consolidation, hub location
- Public utilities (water, electricity, gas, waste) distribution planning
- Telecomm network design
  - Long-distance, local access, ring, wireless
- VLSI, circuit design
- Distributed database location
- Production planning, process design (e.g., chemical)
- Airline operations planning (e.g., fleet assignment)
- Railroad blocking
- Marketing models: Product positioning
- Biology, genetics?

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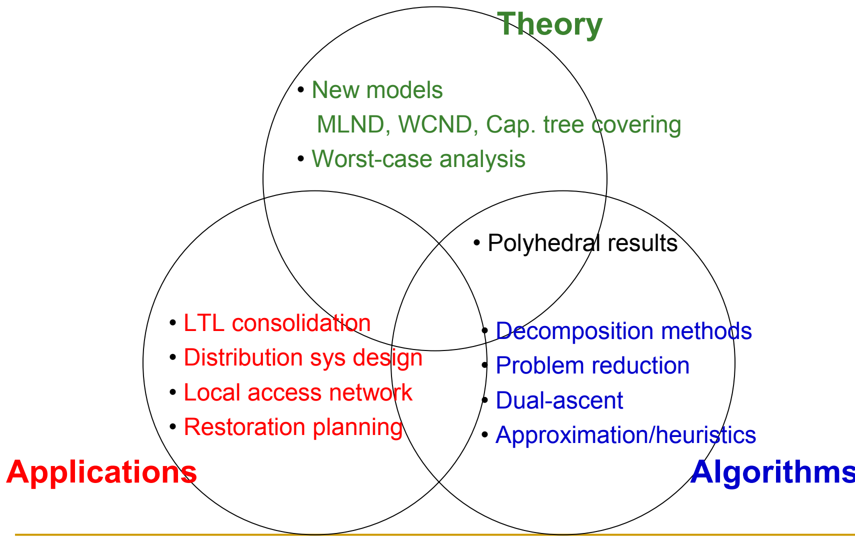
Research perspectives



Basic Network Design model

- No additional routing requirements or design restrictions
- Each commodity k has single origin O_k and single destination D_k
  - Scale all demands to 1 unit
  - Define x_{ij}^k as *fraction* of commodity k's demand flowing from node i to node j
  - Define c_{ij}^k as cost of routing all units of commodity k from node i to node j
- Assume directed graph

Arc flow formulation for Basic model

$$\text{Minimize} \sum_{(i,j)\in E} F_{ij} z_{ij} + \sum_{k\in K} \sum_{(i,j)} c_{ij}^k x_{ij}^k$$

Flow conservation $$\sum_j x_{ij}^k - \sum_j x_{ji}^k = \begin{cases} +1 & \text{if } i = O_k \\ -1 & \text{if } i = D_k \\ 0 & \text{otherwise} \end{cases} \qquad \forall k \in K$$

Forcing constraints $$x_{ij}^k \le z_{ij} \qquad \forall (i,j) \in E, k \in K$$

Nonnegativity $$x_{ij}^k \ge 0 \qquad \forall (i,j) \in E, k \in K$$

Integrality $$z_{ij} \in \{0,1\} \qquad \forall (i,j) \in E$$

Some Model Extensions

- **Node costs**
- **Piecewise-concave costs** (economies of scale)
- **Demand selection**
- **Uncertainty** (limited versions)

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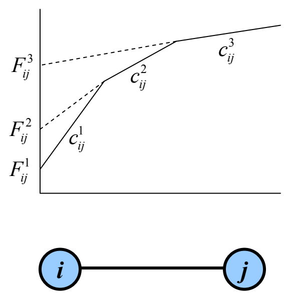
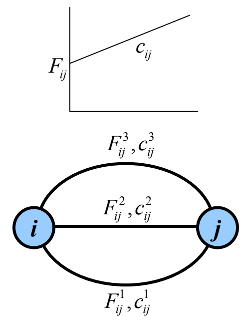
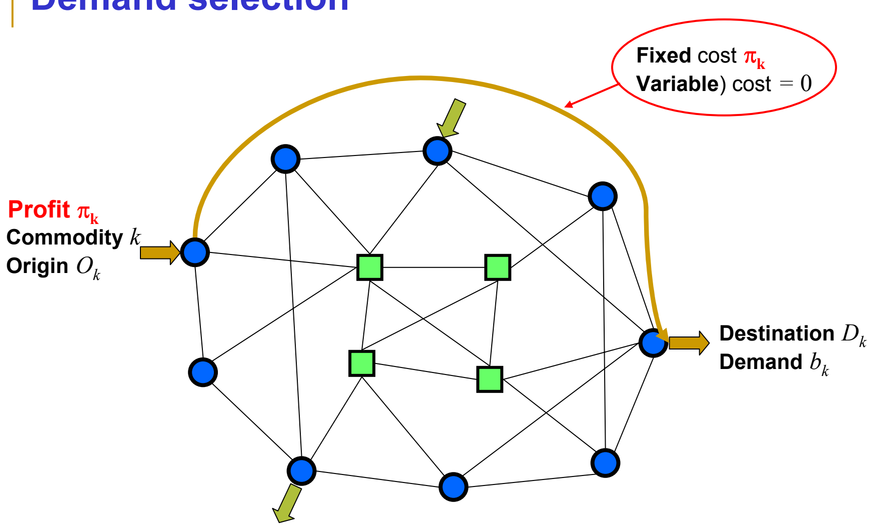
Economies of scale (piecewise linear, concave costs)





Demand selection



DC-to-Store Delivery Fee Table

Delivery Weight (pounds)

Delivery Distance (miles)	0-1000 lbs	1000-4000	4000-10000	> 10,000 lbs
0-30 miles	$40	$70	$100	$140
30-70 miles	$60	$90	$120	$160
> 70 miles	$70	$100	$140	$200

store

Special Cases

- **Shortest Path problem**
  - Only one commodity
  - Fixed cost = arc length; variable cost = 0
- **Minimum Spanning Tree problem**
  - Single source (root node)
  - One commodity each to all other nodes
  - Fixed cost = arc length; variable cost = 0
- **Steiner Tree problem**
  - Same as MST except some nodes are transshipment nodes (no corresponding commodity)

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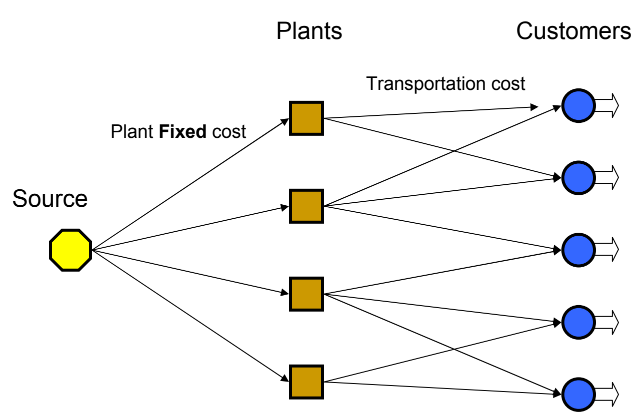
Special cases (continued)

- **Facility Location problem**
- **Dynamic Lot-sizing problem**
- **Traveling Salesman problem**

Facility Location Problem



Bipartite network, single source

Problem complexity

- Model formulation size grows very rapidly with problem size
  - Problem with 100 nodes, 2000 arcs (sparse), 2000 commodities has 4 million variables
- Problem is NP-hard since it generalizes several known difficult problems

Solution approaches

- **Exact** methods
  - Decomposition (e.g., Lagrangian relaxation, Bender's decomposition, column generation)
  - Polyhedral methods (branch-and-cut)
- **Approximate** methods
  - Optimization-based (e.g., dual ascent, primal-dual)
  - Solving restricted (easy) versions
  - Local search & improvement methods

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Dual Ascent principle

- **Approximately solve dual of LP relaxation**
  - By iteratively adjusting the dual multipliers
  - Exploit special problem structure
- **Dual solution provides**
  - *Lower bound* on optimal value (performance guarantee)
  - Starting *feasible solution* for heuristic improvement procedure
  - *Problem reduction* opportunity

Dual Ascent success stories

- Assignment problem (Fisher)
- Steiner Tree problem (Wong)
- Uncapacitated Network Design (BMW)
- Multi-level Network Design (BMM)
- Survivable Network Design (Raghavan)
- Other optimization problems

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LP relaxation of Basic model

$$\text{Minimize} \quad \sum_{(i,j)\in E} F_{ij} z_{ij} + \sum_{k\in K} \sum_{(i,j)\in E} c_{ij}^k x_{ij}^k$$

Dual variables

Flow conservation $\sum_j x_{ij}^k - \sum_j x_{ji}^k = \begin{cases} +1 & \text{if } i = O_k \\ -1 & \text{if } i = D_k \\ 0 & \text{otherwise} \end{cases} \quad \forall i \in N, k \in K$ $\quad v_i^k$

Forcing constraints $x_{ij}^k \le z_{ij} \quad \forall (i,j) \in E, k \in K$ $\quad w_{ij}^k$

Nonnegativity $x_{ij}^k, z_{ij} \ge 0 \quad \forall (i,j) \in E, k \in K$

Integrality $z_{ij} \in \{0,1\} \quad \forall (i,j) \in E$

Dual problem and its special structure

Define $d_{ij}^k = c_{ij}^k + w_{ij}^k$

$$\text{Maximize} \quad \sum_{k\in K} v_{D_k}^k$$

Node potentials $v_j^k - v_i^k \le c_{ij}^k + w_{ij}^k \quad \forall (i,j) \in E, k \in K$

Fixed cost allocation $\sum_{k\in K} w_{ij}^k \le F_{ij} \quad \forall (i,j) \in E$

Non-negativity $v_i^k, w_{ij}^k \ge 0 \quad \forall (i,j) \in E, k \in K$

Given values of w_{ij}^k satisfying the FC allocation constraint, for each commodity k, $v_{D_k}^k \triangleq L_k$ is length of shortest path from O_k to D_k using $d_{ij}^k = c_{ij}^k + w_{ij}^k$ as arc lengths

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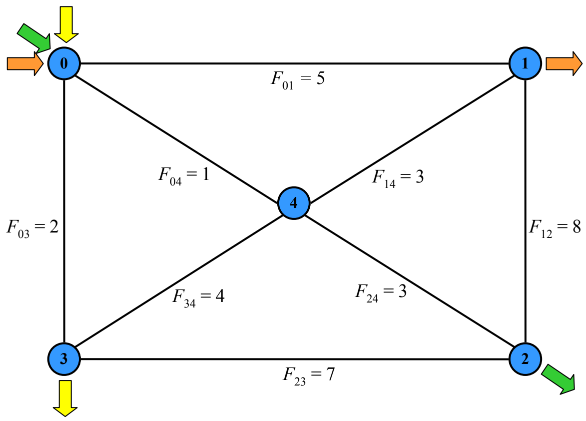
Dual Ascent idea

- Starting with $\boldsymbol{w} = \mathbf{0}$
  - For each commodity k, selectively increase certain w_{ij}^k values—by allocating the arc fixed cost F_{ij}—to increase the value of the shortest path length L_k
  - Stop when no more increases possible.
- **Definitions**
  - $R_{ij} = F_{ij} - \sum_k w_{ij}^k$ = ***Remaining fixed cost*** on edge (i, j)
  - Edge (i, j) is ***tight*** for comm. k if it lies on the shortest O-D path for comm. k
  - $S_{ij}^k = (v_j^k - v_i^k) - d_{ij}^k$ = ***Slack*** on edge (i, j) for comm. k

Labeling procedure

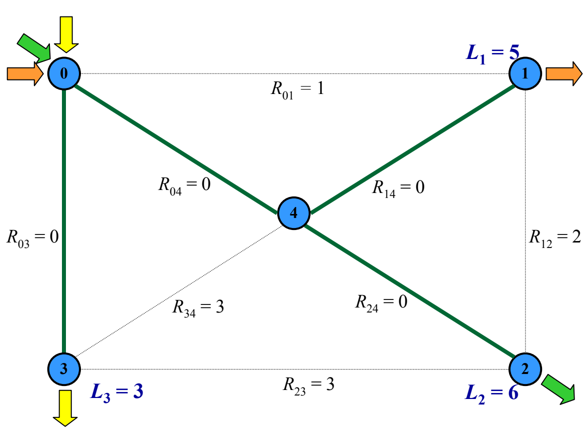
- Similar to Djikstra's shortest path algorithm
- For each **commodity**:
  - Find current shortest paths (*tight* arcs)
  - Define a cutset defined by *labeled* nodes (incl. destination)
  - Increase shortest path length by minimum of
    - *Remaining fixed cost* among *tight* arcs in cutset
    - *Slack* among *loose* arcs in cutset
  - Update dual values, labels
    - Allocate fixed costs for tight arcs,
    - increase dual objective function value, and
    - label additional nodes (if remaining fixed cost is binding)
- Stop when origin node is labeled for all commodities

Dual Ascent example



Variable cost $c_{ij} = 1$ for all edges, all commodities

Final dual solution



Final dual value = 14

Fully allocated edge

Properties of Dual Ascent procedure

- Generalizes
  - Edmond's directed spanning tree algorithm
  - Erlenkotter's DUALOC facility location procedure
  - Wong's Steiner Tree algorithm
- Pseudo-polynomial
- Property of final dual solution
  - At termination, network consisting of "Fully allocated arcs" is feasible for the original problem, i.e., this design is guaranteed to contain at least one O-D path for every commodity

Dual Ascent outputs

- **Lower bound**
  - Final dual value is a lower bound on optimal value of network design problem
- **Heuristic solution**
  - Using feasible design from dual ascent procedure as a starting solution, apply local improvement procedure (e.g., Add/Drop or interchange heuristic)
- **Problem reduction**
  - Eliminate edges based on dual solution

Problem reduction

- Let
  - Z_D = Final dual objective value
  - Z^H = Cost of heuristic solution
  - R_{ij} = Remaining fixed cost of edge (i, j)
- If $R_{ij} > (Z^H - Z_D)$, then edge (i, j) cannot belong to any optimal solution
  ➔ eliminate edge (i, j), and re-apply dual ascent

Dual Ascent enhancements

- Use complementary-slackness conditions from final dual solution to fix w-values, and re-apply dual ascent procedure to possibly improve lower bound
- Test alternate commodity sequencing schemes
- Add methods to reallocate w-values (versus only increasing these values)
- Modify method to incorporate additional constraints

Test problems

Network Dimensions for Undirected Test Problems

Problem Number	No. of Nodes	No. of Arcs	No. of Commodities	Number of Variables in Formulation P_1: Integer	Number of Variables in Formulation P_1: Continuous
1	20	80	380	80	60,800
2	25	100	600	100	120,000
3	30	130	870	130	226,200
4	35	150	1190	150	357,000
5	40	400	1560	400	1,248,000
6	45	500	1980	500	1,980,000
Complete Networks					
7	15	105	210	105	44,100
8	20	190	380	190	144,400
9	25	300	600	300	360,000
10	30	435	870	435	756,900
11	35	595	1190	595	1,416,100

Computational results

Dual-Ascent Performance for Asymmetric Test Problems (all test problems have FC/VC ratio = 10.0)

Problem Size	% Gap[a]	CPU Time: Total[b]	CPU Time: % Ascent[c]	No. of Arcs Deleted	No. of Ascent Cycles
1	2.15	12.33	53.6	9	4
2	1.30	16.76	51.0	16	3
3	2.21	23.48	42.8	0	2
4	1.70	33.62	35.7	0	2
5	2.48	141.67	40.4	0	2
6	3.34	285.32	29.6	0	2
7	3.54	13.35	63.8	27	5
8	4.02	19.47	61.4	0	2
9	4.03	48.45	60.9	0	2
10	3.79	106.40	52.3	0	2
11	3.97	113.65	47.5	0	2

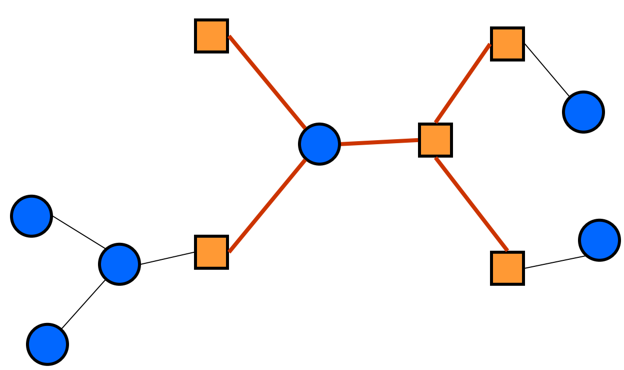
[a] % gap = (upper bound − lower bound)/lower bound.
[b] CPU time in seconds on an IBM 3083 (model BX).
[c] Proportion of CPU time required for the ascent procedure.

Multi-level Network Design (MLND)

- *Nodes* are classified by *level of importance* into different *types*
- Correspondingly, we consider different *facility* (edge) *types*
- Higher-level nodes must be interconnected by higher grade facilities; this sub-network may optionally include lower-level nodes

- Higher grade facilities are more expensive
- Special case: **Two-level Network Design (TLND)**
  - ***Primary*** nodes ***P***, ***Secondary*** nodes ***S***
  - Two types of facilities: *primary* and *secondary* with fixed costs a_{ij} and b_{ij}

Multi-level Network Design



TLND Problem formulation

$$\text{Minimize} \sum_{(i,j)\in E} a_{ij} y_{ij} + \sum_{(i,j)\in E} b_{ij} z_{ij} + \sum_{k\in K} \sum_{(i,j)} c_{ij}^k f_{ij}^k$$

Flow conservation $$\sum_j x_{ij}^k - \sum_j x_{ji}^k = \begin{cases} +1 & \text{if } i = O_k \\ -1 & \text{if } i = D_k \\ 0 & \text{otherwise} \end{cases} \quad \forall k \in K$$

Primary forcing constraints $x_{ij}^k \le y_{ij}$ $\quad \forall (i,j) \in E, k \in P$

Secondary forcing constraints $x_{ij}^k \le y_{ij} + z_{ij}$ $\quad \forall (i,j) \in E, k \in S$

Nonnegativity $x_{ij}^k \ge 0$ $\quad \forall (i,j) \in E, k \in K$

Integrality $y_{ij}, z_{ij} \in \{0,1\}$ $\quad \forall (i,j) \in E$

Special cases and complexity of TLND

- **Steiner tree** problem
  - b_{ij} = 0 for all edges (i,j)
- (Hierarchical) **Path-tree** problem
  - Number of primary nodes $|P| = 2$
- **Proportional-cost TLND** problem
  - $a_{ij} = r\, b_{ij}$ for all edges (i,j)

The TLND problem is NP-hard even if $|P| = 2$ and either costs are proportional or $a_{ij} = 1$ and $b_{ij} = 0$ or 1

Illustrative Worst-case result for TLND problem

Consider two alternative heuristics

- **Secondary Extension (SE)** method
  - Connect primary nodes via Steiner tree
  - Extend this tree with secondary facilities to span remaining secondary nodes
- **Primary Upgrade (PU)** method
  - Connect all nodes via min spanning tree, with secondary facilities
  - Upgrade facilities on induced primary subtree
- SE method is near-optimal when secondary costs are small, whereas PU method is near-optimal when secondary costs are close to primary costs
- → *Hybrid* method: select the better of the SE and PU solutions

Theorem: For the proportional-cost TLND problem, if ρ denotes the worst-case ratio of the Steiner tree solution in the SE method, then the Hybrid method has worst-case ratio of $4/(4-\rho)$, and the bound is tight

MLND Solution strategy

- Problem pre-processing to eliminate edges, flow variables
- Dual ascent to
  - Generate lower bound
  - Identify feasible solution
  - Reduce the problem (fix variables)
- Heuristic improvement of dual-based solution

MLND problem: Computational results

Table 1 **Effect of Preprocessing and Problem Size***

Euclidean and random costs

	Without Preprocessing			With Preprocessing				
Problem Category†	Average % Gap††	Average Ascent Time	Average Add-Drop Time	Avg % of Primary Nodes Aggregated	Average % Gap††	Average Set-up Time	Average Ascent Time	Average Add-Drop Time
(50/100, 500, *EP*)	0.21	11	4	40	0.21	2	8	3
(50/100, 500, *EN*)	0.15	6	3	53	0.13	2	4	2
(80/200, 1,000, *EP*)	0.08	56	59	39	0.08	7	48	57
(80/200, 1,000, *EN*)	0.48	66	75	35	0.48	7	53	71
(300/400, 2,000, *EP*)	0.01	842	189	77	0.02	25	115	82
(300/400, 2,000, *EN*)	0.00	685	234	74	0.00	25	188	212
(50/100, 500, *RP*)	0.06	6	2	53	0.07	2	4	2
(50/100, 500, *RN*)	0.09	5	4	53	0.07	2	3	3
(80/200, 1,000, *RP*)	0.89	2,710	42	40	0.89	7	1,900	40
(80/200, 1,000, *RN*)	0.68	274	150	43	0.65	7	124	94
(300/400, 2,000, *RP*)	0.02	307	72	71	0.02	26	131	57
(300/400, 2,000, *RN*)	0.01	1,286	311	73	0.01	25	351	223

* All statistics averaged over 3 problem instances. Computational times in seconds on IBM 4381.

† *E* denotes Euclidean cost structure; *R* denotes Random cost structure; *P* denotes Proportional costs; *N* denotes Nonproportional costs.

†† % gap = (best upper bound − best lower bound)/best lower bound.

Importance of good model formulation

- Dual ascent methods are successful when problem has special structure and *model formulation is tight*

Tight problem formulations

- Obtained by adding valid inequalities (ideally, facets) and expanding the set of variables
- May vastly increase problem size, but help to:
  - Generate good lower bounds
    - Better guarantees for heuristic solution quality
  - Improve algorithmic performance (lesser enumeration needed in exact algorithms)
  - Identify better heuristic solutions
- Use iterative (cutting plane, column generation) methods to cope with larger problem size

Example: Forcing constraints in Network Design models

Assume undirected edges

Aggregate forcing constraints

$$\sum_{k\in K} x_{ij}^k \leq |K| z_{ij} \quad \& \quad \sum_{k\in K} x_{ji}^k \leq |K| z_{ij} \qquad \forall (i,j)\in E$$

Disaggregate forcing constraints

$$x_{ij}^k \leq z_{ij} \quad \& \quad x_{ji}^k \leq z_{ij} \qquad \forall (i,j)\in E, k\in K$$

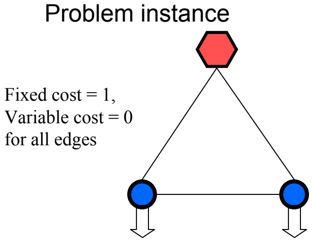
Bidirectional forcing constraints (when flow costs are same for all comm.)

Consider pairs of commodities *k* and *h* with same origin or destination

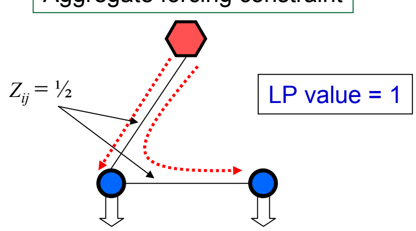
$$x_{ij}^k + x_{ji}^h \leq z_{ij} \qquad \forall (i,j)\in E; k,h\in K \text{ with } O_k = O_h \text{ or } D_k = D_h$$

Example: Strength of forcing constraints

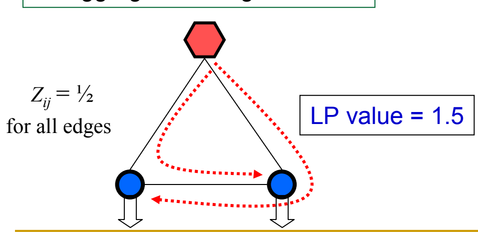
Problem instance



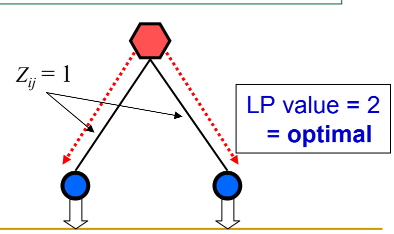
Aggregate forcing constraint



Disaggregate forcing constraint



Bi-directional forcing constraint



Learnings

- Network design problems are **important** for strategic, tactical, and operational planning of distribution systems, but they are **challenging**
- Develop **tailored** algorithms that exploit the problem's **special structure**
- **Strong problem formulations** are critically important
- Combine **multiple techniques**—problem preprocessing, decomposition, efficient subproblems, iterative model enhancement, problem reduction, heuristic search and improvement

Network Design extensions

Network Design Model Variant	Characteristics
Capacitated Network Design (CND)	Capacitated edges, no bifurcation of flows
Hop-constrained Network Design (HCND)	# of edges on flow path must not exceed specified maximum
Weight-constrained Network Design (WCND)	Total weight (e.g. delay) on flow path must not exceed specified maximum
Network Loading (NL)	Discrete set of available edge capacities; no flow costs
Multi-level Network Design (MLND)	Multiple node types; higher level nodes require higher grade facilities (edges)
Survivable Network Design (SND)	Require disjoint alternate paths between node pairs
Network Restoration (NR)	Non-simultaneous flow; flow created by failure of edge