Designing Robust Supply Chains

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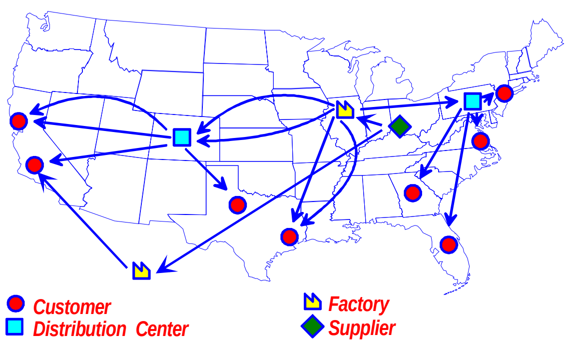
Overview

- Supply chain strategic design problem
- Robustness and flexibility
- Hierarchical stochastic design procedure
- Computational experience
- Conclusions

Supply Chain Network



Supply Chain Design Objectives

- Cost minimization
- Return on investment maximization
- Profit maximization
- Robustness
- Responsiveness
- Flexibility
- Usually conflicting

Strategic Supply Chain Design Profit Maximization Objective

- The profit maximization objective is based on the risk analysis of the maximized net present value of the time-discounted net cash flows created by executing logistics missions over the planning horizon.

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Supply Chain Strategic Design Problem

- Multicommodity, multiperiod, multi-echelon, multi-country capacitated network flow problem (nodes, arcs)
- Decision variables
  - Binary status variables for facility, technology, and machines
  - Continuous flow & storage variables
- Objective function, constraint matrix, right-hand side all can be stochastic

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Design of Robust Supply Chains

- Change in the mission and data is inevitable, but only techniques are sensitivity and scenario analysis and simulation
- No scientific analysis or design methodology for large SC problems
- Needed are measures of
  - Robustness, flexibility, stability

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Robustness and Flexibility

- Relative robustness, Kouvelis (1997)

$$\max_{s \in S} \left\{ \frac{z_s(x_R) - z_s^*(x_s^*)}{z_s^*(x_s^*)} \right\}$$

- Flexibility, Beamon (1998)
  - Unused capacity in a configuration
- Stability (?) for multi-period planning

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Hierarchical Stochastic Design Algorithm

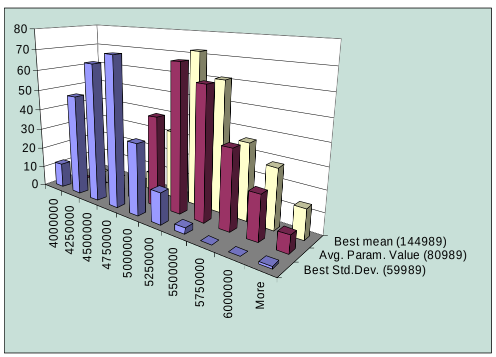
- Select a limited number M of feasible supply chain configurations
  - Each based on N scenarios
- For each configuration
  - Sample N' scenarios
  - Solve linear network flow problems
  - Compute expected value and variance
- Select "best" configuration
  - Weighted objective or efficiency frontier

Hierarchical Two-stage Formulation

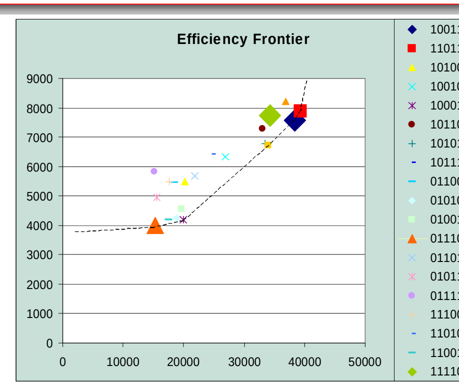
$$\begin{aligned} Min\quad & cx + dy \\ s.t.\quad & Ex + Fy \le h \\ & Hx \le g \\ & x \in \{0,1\}, y \ge 0 \end{aligned}$$

$$\begin{aligned} Min\quad & cx + E\left[Q(x,\xi)\right] \\ s.t.\quad & Hx \le g \\ & x \in \{0,1\} \end{aligned} \qquad Q(x,\xi) = \begin{aligned} Min\quad & dy \\ s.t.\quad & Fy \le h - Ex \\ & y \ge 0 \end{aligned}$$

Second Stage Profit Distributions (Medium Example)



Multi-objective Criteria and Efficiency Frontier (Medium)



Stochastic Strategic Supply Chain Solution Algorithm Challenges

- *Very uncertain data*
  - *Requires large number of scenarios*
  - *Sample Average Approximation (SAA)*
- *Very large MIP formulation, but with significant structure*
  - *Primal Benders L-Shape decomposition converges too slowly*
  - *Acceleration techniques for Benders*

Sample Average Approximations (SAA)

$$E\left[Q(x,\xi)\right] \approx \frac{1}{N}\sum_{i=1}^{N} Q\left(x,\xi^{i}\right)$$

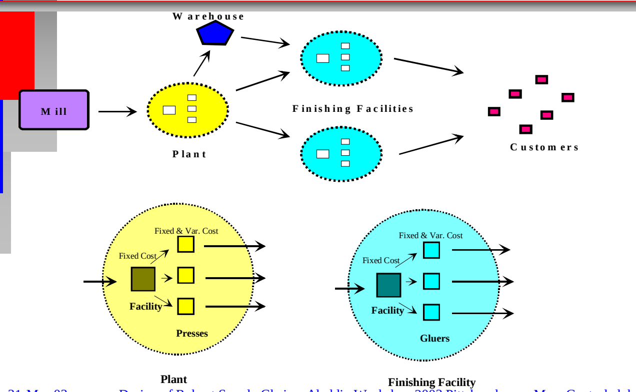
$$SD\left[Q(x,\xi)\right] = \sqrt{Var\left[Q(x,\xi)\right]}$$

$$Var\left[Q(x,\xi)\right] \approx \frac{1}{N}\sum_{i=1}^{N} Q^{2}\left(x,\xi^{i}\right) - \left\{\frac{1}{N}\sum_{i=1}^{N} Q\left(x,\xi^{i}\right)\right\}^{2}$$

Acceleration Techniques for Benders Decomposition

1. *Logistics constraints*
   - *Echelon capacity, source capacity*
2. *Trust region*
   - *Increasing number of binary changes*
3. *Knapsack upper bounds (min. case)*
4. *Primal feasible heuristic*
   - *Optimal solution for given facilities*
5. *Cut strengthening*
   - *Maximize dual variables with zero coeff.*

Two Stage Supply Chain With Machine Resources



Domestic Case Formulation Characteristics

Problem Statistics	N=1	N=20	N=40	N=60
Constraints	7,822	156,440	312,880	469,320
- Inequality constraints	3,498	69,960	139,920	209,880
- Equality constraints	4,324	86,480	172,960	259,440
Variables	21,052	418,380	836,620	1,254,860
- Continuous variables	20,912	418,240	836,480	1,254,720
- Integer (binary) variables	140	140	140	140

N = Number of scenarios in master problem

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Acceleration Techniques Performance: Quality (Gap)

	Original	1	2	3	4	5
1st Gap	> 100%	31%	> 100%	60%	> 100%	> 100%
10th Gap	60%	8%	40%	5%	60%	9%

	Original	1+2	1+3	1+4	1+5	1+2+3
1st Gap	> 100%	31%	31%	31%	31%	31%
10th Gap	60%	0.7%	0.1%	0.08%	0.5%	0.2%

	Original	2+3	1+3+4	1+2+3+4	1+2+3+5	All
1st Gap	> 100%	60%	31%	31%	31%	31%
10th Gap	60%	3%	0.01%	0.01%	0.06%	0.01%

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Acceleration Techniques Performance: Efficiency (Run Time)

N=20

	Original	1	2	3	4	5
Time	> 4,000	> 4,000	> 4,000	> 4,000	> 4,000	> 4,000
Iteration	> 30	> 30	> 30	> 30	> 30	> 30

	Original	1+2	1+3	1+4	1+5	1+2+3
Time	> 4,000	> 4,000	3,860	2,180	> 4,000	3,600
Iteration	> 30	> 30	26	12	> 30	23

	Original	2+3	1+3+4	1+2+3+4	1+2+3+5	All
Time	> 4,000	> 4,000	1,500	1,380	3,050	1,890
Iteration	> 30	> 30	8	7	19	7

Note: The MIP Optimal using CPLEX needs 2,700 seconds

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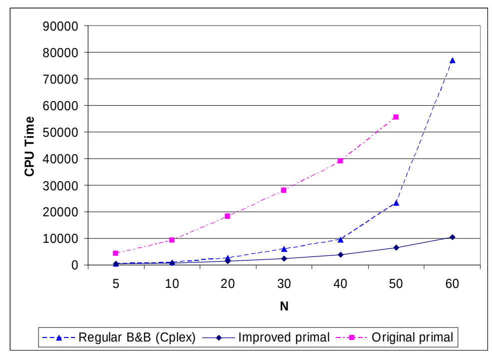
Comparison of Run Times for Solution Algorithms



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Stochastic Solutions Robustness for Domestic Case

N=20, M=20, N'=1000

Configuration	MPP	SS-1	SS-2	SS-3
Average obj. value (in million $)	116.77	111.03	111.03	111.05
Max	173.30	122.57	122.08	122.11
Min	99.02	100.38	100.14	100.10
Range	74.28	22.19	21.93	22.01
Standard Deviation	0.34	0.11	0.11	0.11
Absolute Gap	5.91	0.16	0.17	0.18
Relative Gap	0.066574	0.001454	0.001508	0.001619

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Impact of Variability

Problems	Std. Deviation for customer demand	Std. deviation for all other parameters
Medium variability problem	30%	10%
Low variability problem	15%	5%
High variability problem	40%	20%

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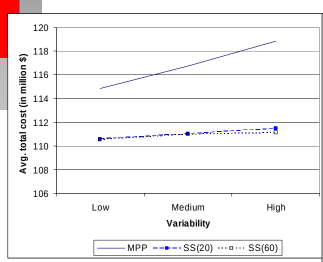
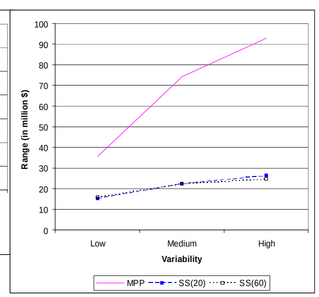
Robustness of Stochastic Solutions for Domestic Case





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Global Case Problem Characteristics

Problem Statistics	*N*=1	*N*=10	*N*=20	*N*=60
Constraints	1,467	14,670	29,340	88,020
- Inequality constraints	402	4,020	8,040	24,120
- Equality constraints	1,065	10,650	21,300	63,900
Variables	6,894	68,310	136,550	409,510
- Continuous variables	6,824	68,240	136,480	409,440
- Integer (binary) variables	70	70	70	70

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Acceleration Techniques Performance: Quality (Gap)

N=20

	Original	1	2	3	4	5
1st Gap	> 100%	> 100%	> 100%	> 100%	> 100%	> 100%
50th Gap	41 %	27 %	18 %	21 %	41 %	29 %

	Original	1+2	1+3	1+4	1+5	1+2+3
1st Gap	> 100%	> 100%	> 100%	> 100%	> 100%	> 100%
50th Gap	41 %	22 %	12 %	27 %	19 %	3 %

	Original	2+3	1+3+4	1+2+3+4	1+2+3+5	All
1st Gap	> 100%	> 100%	> 100%	> 100%	> 100%	> 100%
50th Gap	41 %	5 %	12 %	4 %	< 1 %	< 1 %

Acceleration Techniques for Benders Decomposition

❶ Logistics constraints
- Echelon capacity, source capacity for average demand

❷ Cut disaggregation
- By scenario

❸ Knapsack lower bounds (max. case)

❹ Primal feasible heuristic

❺ Cut strengthening

Acceleration Techniques Performance: Efficiency (Run Time)

N=20

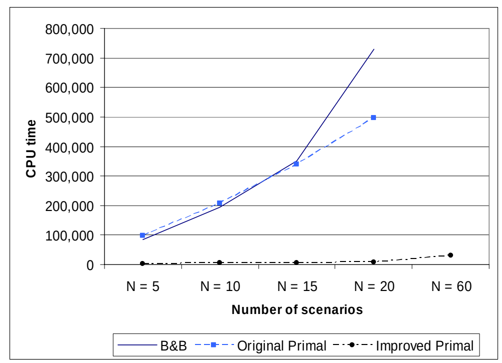
	Original	1	2	3	4	5
Time	>13,000	>13,000	>13,000	>13,000	>13,000	>13,000
Iteration	> 60	> 60	> 60	> 60	> 60	> 60

	Original	1+2	1+3	1+4	1+5	1+2+3
Time	>13,000	11,900	>13,000	>13,000	12,300	10,300
Iteration	> 60	56	> 60	> 60	58	51

	Original	2+3	1+3+4	1+2+3+4	1+2+3+5	All
Time	>13,000	>13,000	>13,000	10,300	9,800	9,800
Iteration	> 60	> 60	> 60	51	45	45

Note: The MIP Optimal using CPLEX needs 730,500 seconds)

Comparison of Run Times for Solution Algorithms



Stochastic Solutions Robustness for the Global Case

N=60, M=10, N'=1000

	MPP	SS
Average NCF	51.021	54.095
Std. Deviation	0.127	0.119
Max NCF	66.996	68.063
Min NCF	31.355	46.531
Range	35.641	21.533
Absolute Gap	3.166	0.092
Relative Gap	0.058425	0.001694

(in million $, except the relative gap)

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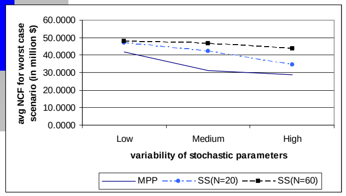
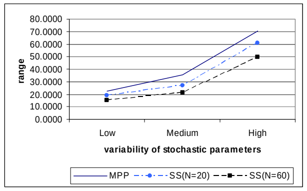
Robustness of Stochastic Solutions for Global Case





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Computational Experiment Conclusions: Robustness

- *Designing for average parameters yields a dominated configuration*
  - *Worse mean and standard deviation*
  - *Difference larger for more variable data*
- *More scenarios yield more robust solution*
- *Statistics stable after 500 samples*
- *Report multi-criteria solutions diagram*

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Numerical Experiment Conclusions: Algorithms

- *Significant smaller times for accelerated primal decomposition*
  - *Better than B&B, original primal decomposition*
  - *Dual decomposition worse ??*
- *Combination of acceleration techniques provides time reduction*
- *Conjecture: Bill of Material makes problem much harder to solve*

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Conclusions

- Good definitions and measures for flexibility and robustness are lacking
- Current methodology is deterministic design and sensitivity or few-scenario analysis
- Many-scenario solutions are more robust
- Only accelerated hierarchical design algorithm fast enough

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Design Tools for the Third Millennia

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Strategic Supply Chain Design Modeling Challenges

- Integrated models are large and complex
  - More tactical effects (seasonal, inventory)
- Multi-objective performance measures
  - Cost/profit, robustness, flexibility, and responsiveness tradeoffs
- Strategic design as a continuous effort
  - Models, data, algorithms
  - Technology transfer and maintenance

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Strategic Supply Chain Design Algorithm Challenges

- Multiple periods, tactical & strategic
  - Periodic and seasonal demand
  - Dynamic strategic systems
- Global financial
  - Taxes and profit realization
  - Local contents, duty drawback, transfer prices
- Stochastic and Multi-criteria
  - Flexibility, robustness, stability

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Thank You
Can I Answer Any Questions?