Near Optimal Online Auctions

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(Joint work with Avrim Blum)

October 28, 2004

Online Auction Problem

Online Auction Problem: [Bar-Yossef, Hildrum, Wu 2002]

- Seller has unlimited supply of an item (e.g., digital good).
- Bidders arrive one at a time, bid $b_1, b_2, \ldots \in [1, h]$
- Auctioneer decides sale price (or reject) before next bidder arrives.
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Conclusion: offer for bidder i based only on prior bids: $b_1, \ldots, b_{i-1}$.

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Goal: $\mathbf{E}[\text{Profit}] \geq \mathrm{OPT}/\beta - \gamma h$.

- $\beta =$ approximation ratio.
- $\gamma h =$ additive loss.

Overview

$\Longrightarrow$ 1. Standard Expert Learning Algorithm (Weighted Majority).

2. Online (Expert) Learning $\Rightarrow$ Online Auction
   Result: $(1-\epsilon)\,\mathrm{OPT} - O(\frac{h}{\epsilon}\log\log h)$, given $[1,h]$.
   [Blum Kumar Rudra Wu 2003]

3. Kalai's Expert Learning Algorithm & Analysis.

4. Modifications of Kalai's Expert Algorithm for Online Auctions.
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5. An Application: Attribute Auctions.

Online Learning

Expert Online Learning Problem:

In round i:

1. Each of k experts propose a strategy.
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Choose expert j's strategy with probability proportional to $2^{s_j/h}$.

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Drawbacks:

- $h \log\log h$ additive loss.
- Must know h in advance.

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Expert Learning via Hallucination

Kalai's Experts Algorithm:

1. (initialization) For each expert, j, hallucinate initial score, s_j, as:

$$h \times \text{ number of heads flipped in a row.}$$

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Definitions: (random variables)

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Lemma: $\mathbf{E}[P_i] \geq \mathbf{E}[M_i]/2$.

Theorem: $\mathbf{E}[P] \geq \mathrm{OPT}/2 - O(h \log k)$.

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- $\mathbf{E}[M] \geq \mathrm{OPT}$ and $\mathbf{E}[H] = O(h \log k)$.

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- $\mathbf{E}[P] = \overbrace{\sum_i \mathbf{E}[P_i] \geq \frac{1}{2} \sum_i \mathbf{E}[M_i]}^{\text{from Lemma}} = \frac{1}{2}\left(\mathbf{E}[M] - \mathbf{E}[H]\right)$.
- $\mathbf{E}[M] \geq \mathrm{OPT}$ and $\mathbf{E}[H] = O(h \log k)$.
- **Result:** $\mathbf{E}[P] \geq \frac{1}{2}\mathrm{OPT} - O(h \log k)$.

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Proof: (after round i)

1. Tally expert "raw scores" (without hallucination) after round i.

2. Re-hallucinate (repeat until one expert is left):

   Flip coin of expert with lowest score

   - **Heads:** add h to expert's score.
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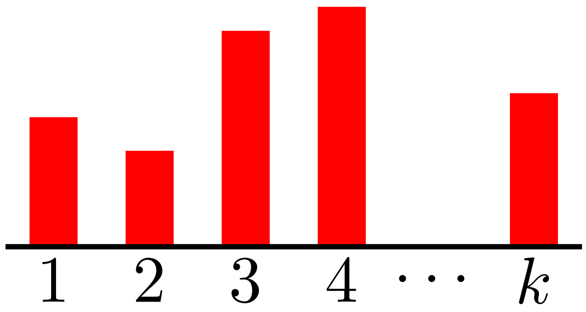

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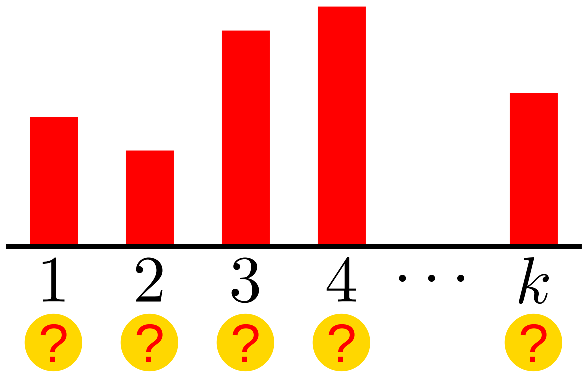

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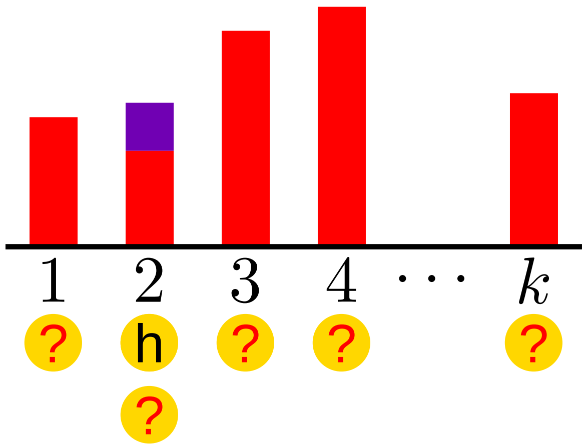

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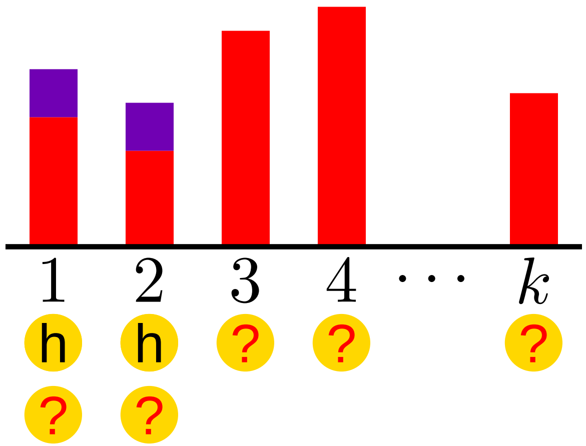

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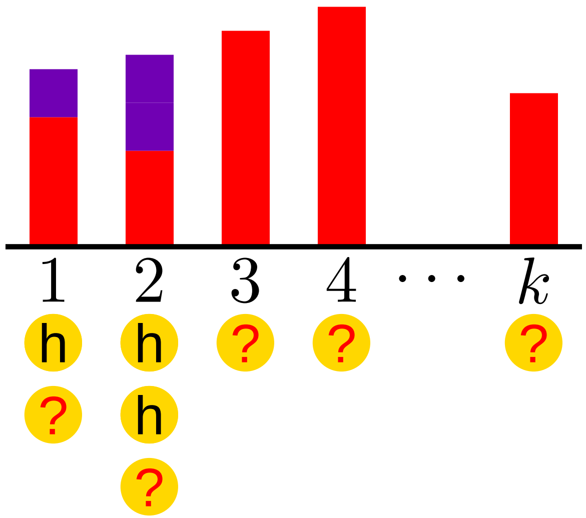

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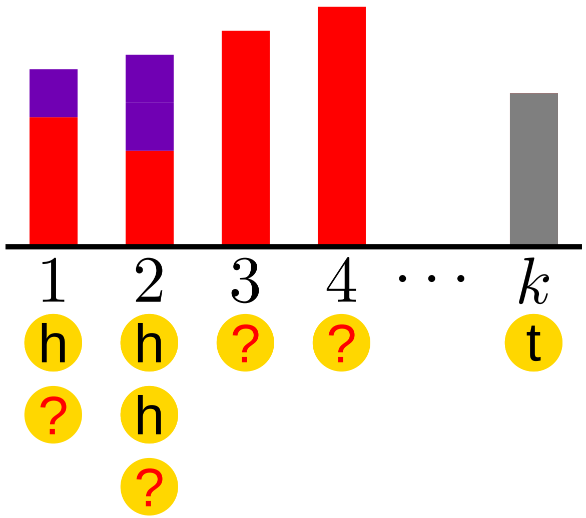

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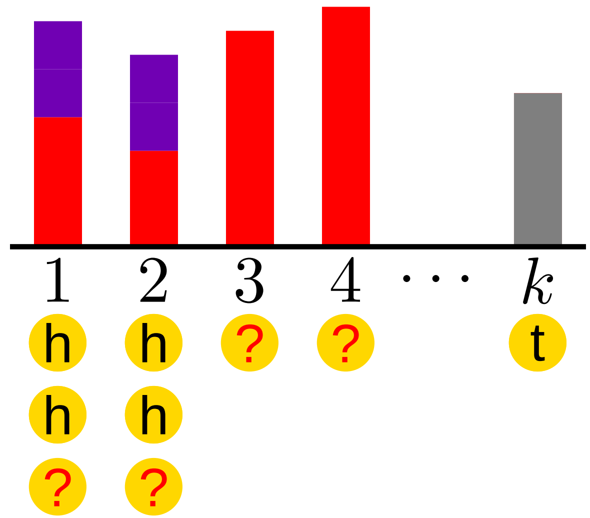

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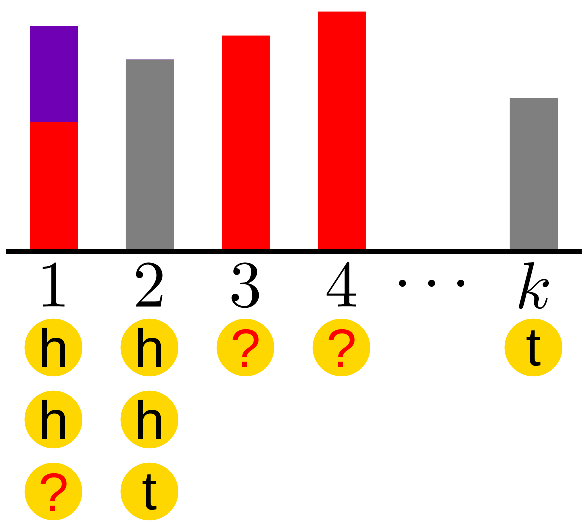

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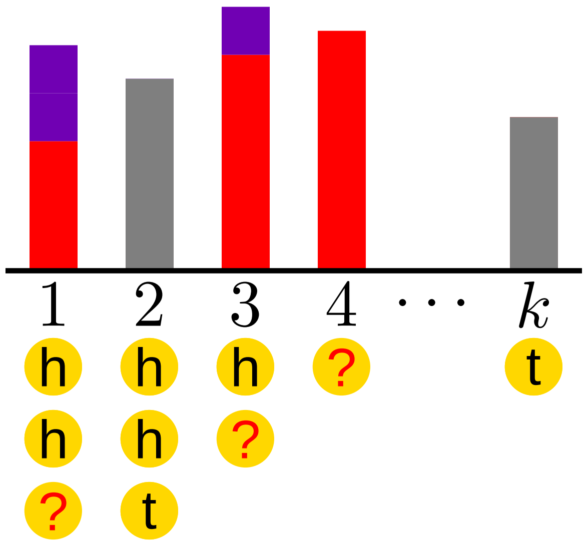

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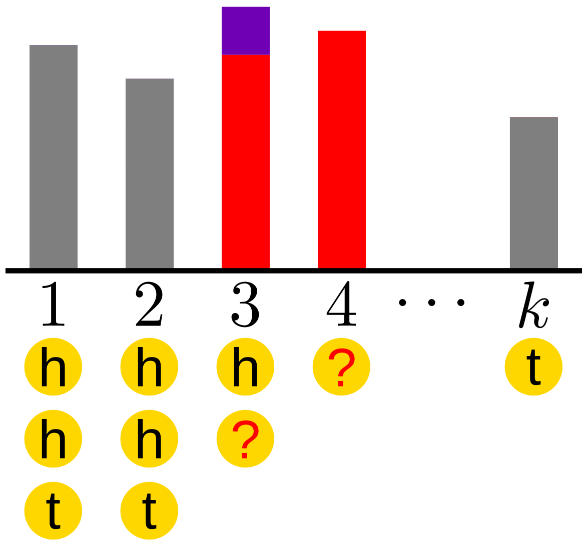

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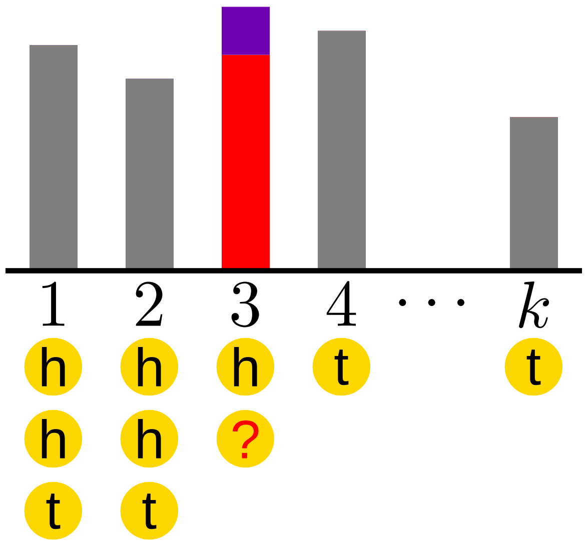

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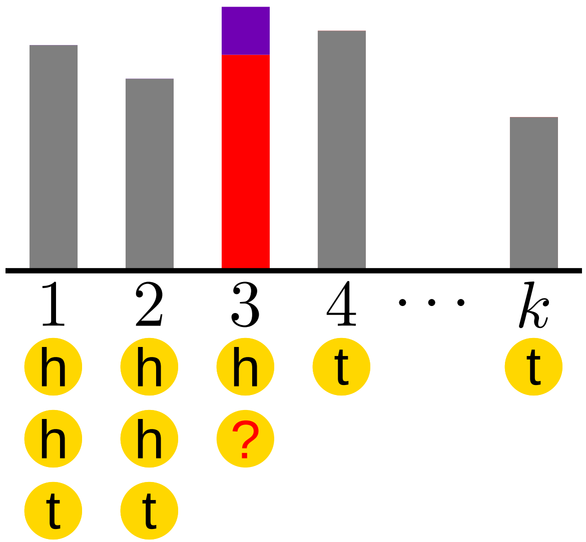
3. Remaining expert j: best and still has coin.



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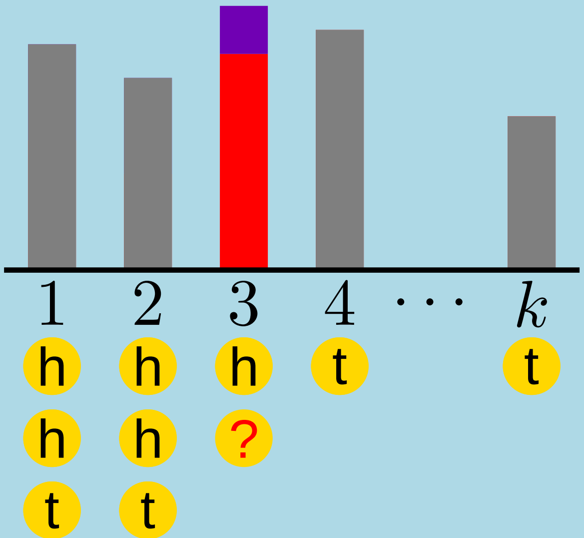

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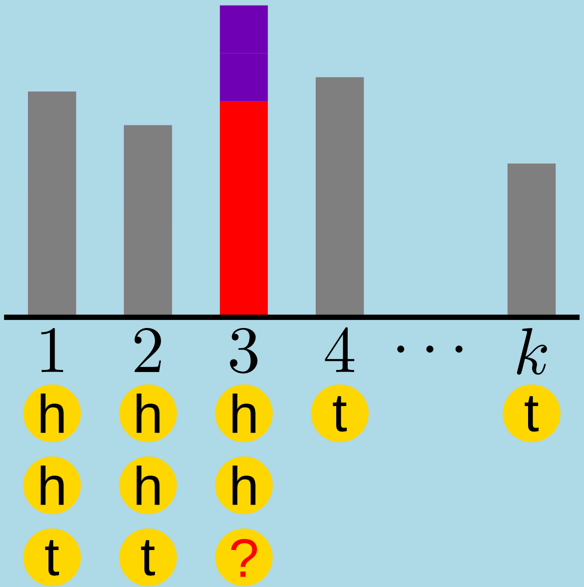

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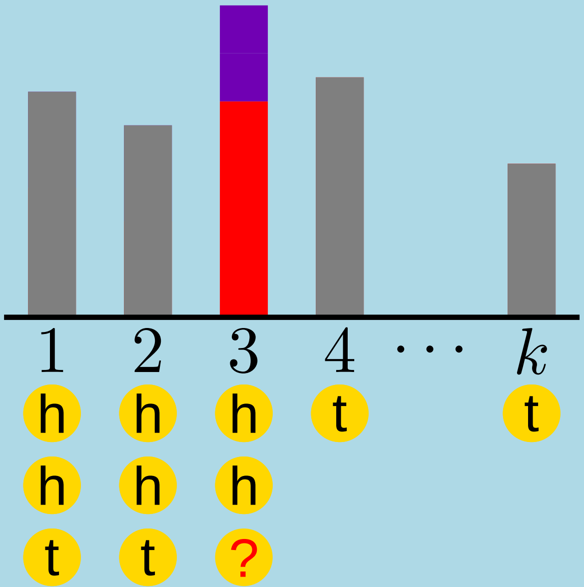

Recall: j's payoff $\leq h$.

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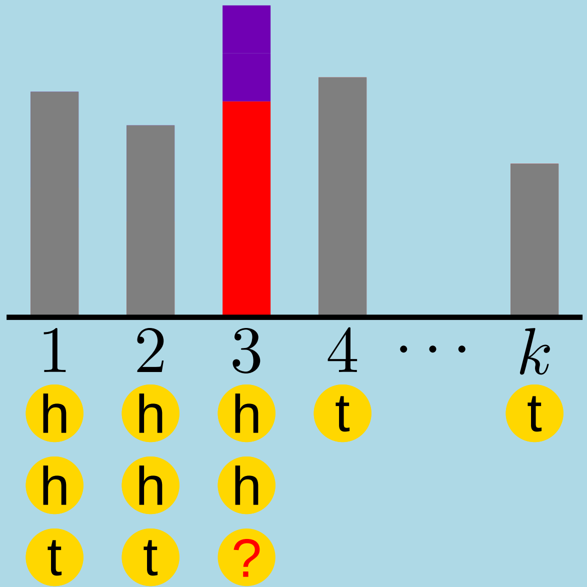

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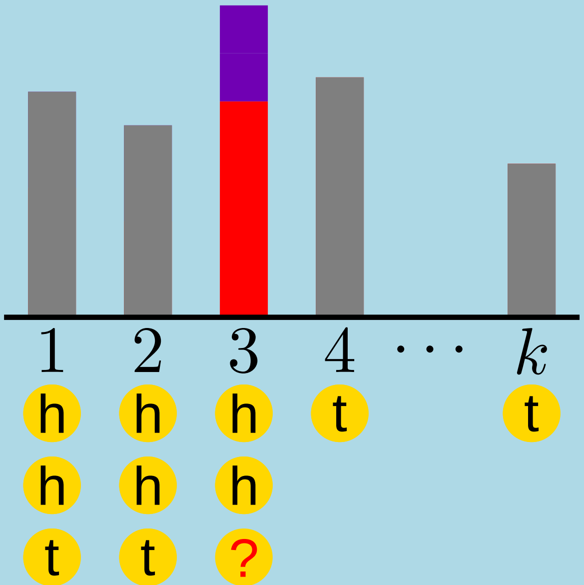

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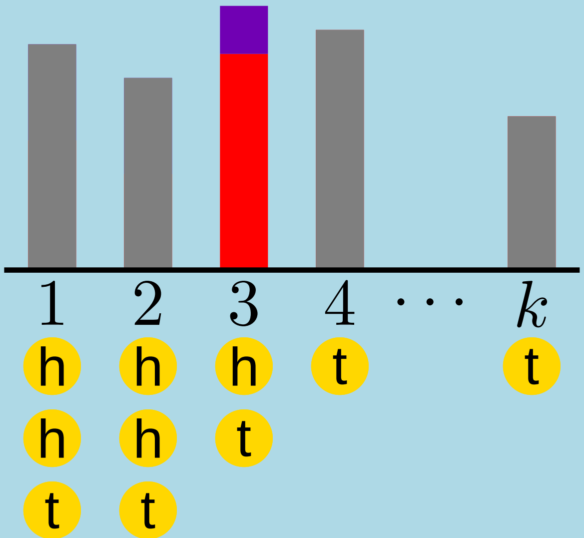

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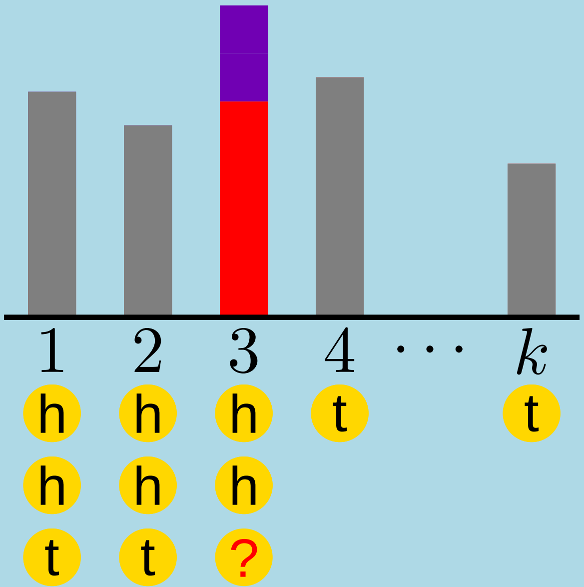

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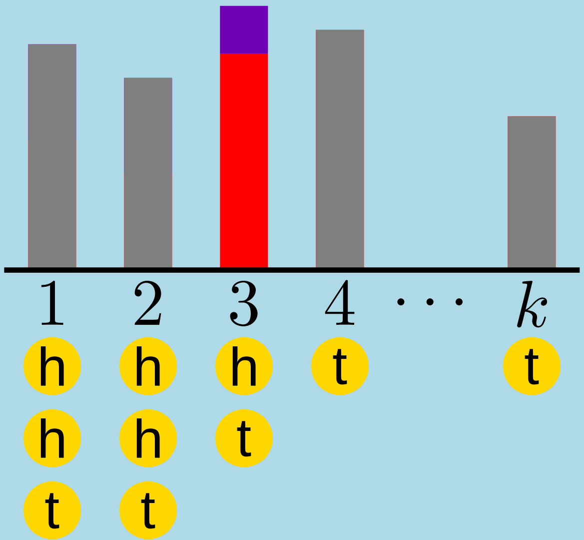

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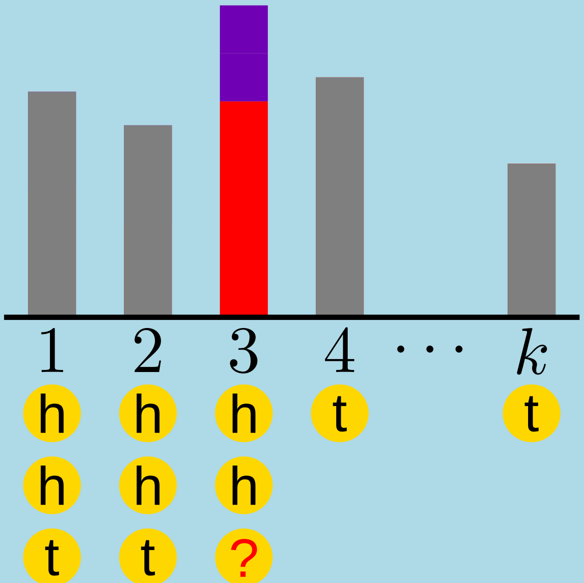

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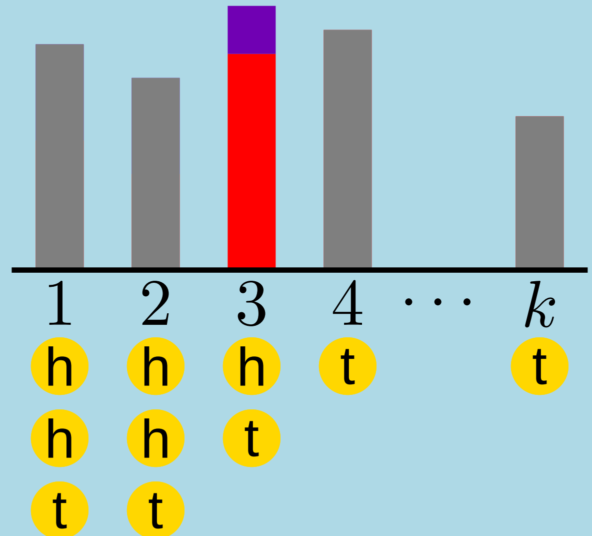

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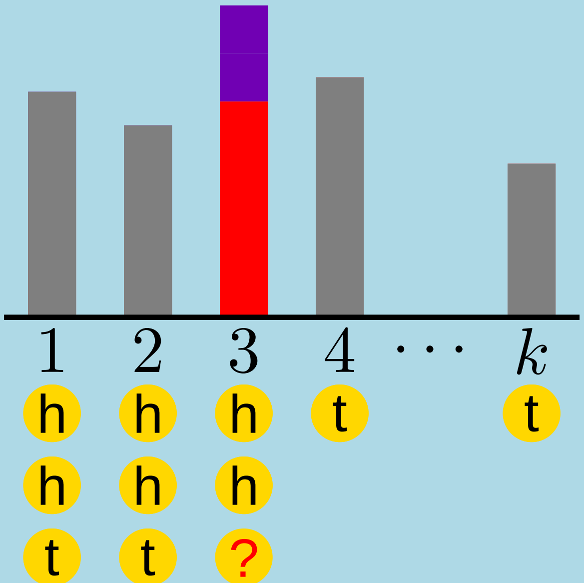

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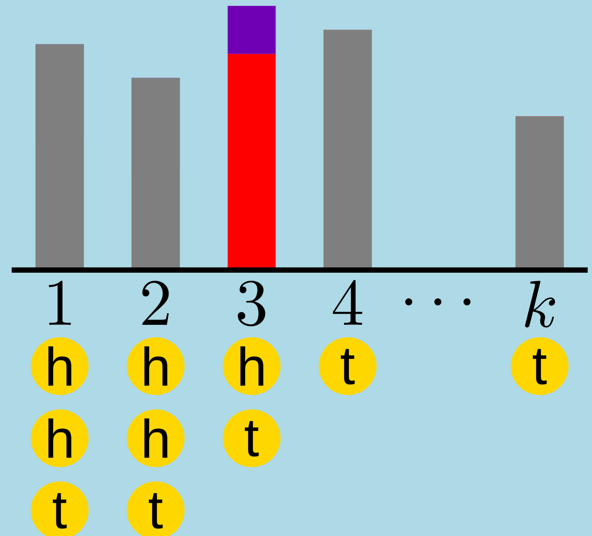

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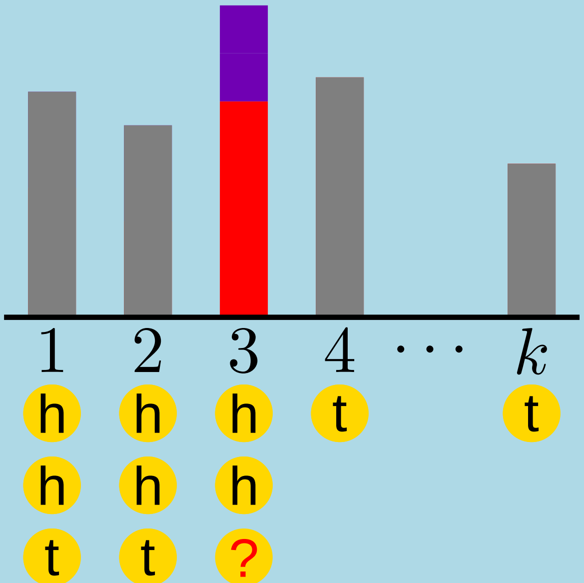

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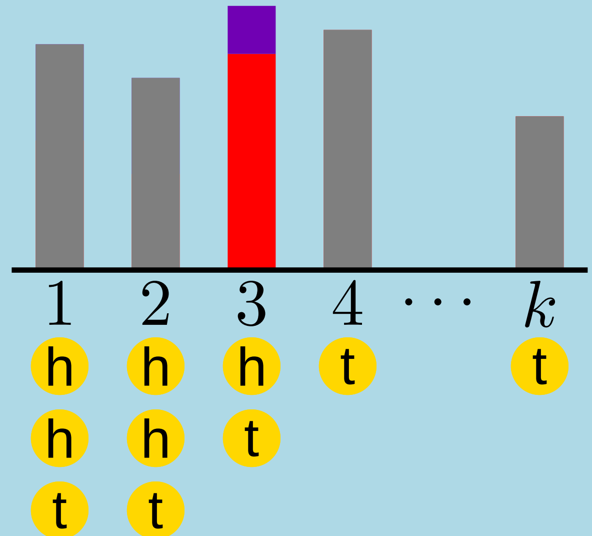

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Proof:

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Given:

- n identical items for sale.
- n indistinguishable bidders.

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What if bidders are distinguishible?

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Truthful mechanism design: For bidder i offer price

$$p_i = f\begin{pmatrix} a_1, & \dots, & a_{i-1}, & a_i, & a_{i+1}, & \dots, & a_n \\ b_1, & \dots, & b_{i-1}, & ?, & b_{i+1}, & \dots, & b_n \end{pmatrix}$$

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Open: Multidimensional Attribute Auctions?

Open: Structured Attribute Auctions?