Traffic Flow Optimization under Fairness Constraints with Lagrangian Relaxation and Cutting Plane Methods

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Flexible Network Design, Bertinoro 2006

Outline

1. Traffic Flow Optimization under Fairness Constraints
   - Motivation
   - The Constrained System Optimum Problem (CSO)
2. Solving the CSO Problem
   - Lagrangian Relaxation to Treat Non-Linearity
   - Proximal-ACCPM: An Interior Point Cutting Plane Method
3. Results
   - Computational Study
   - Summary

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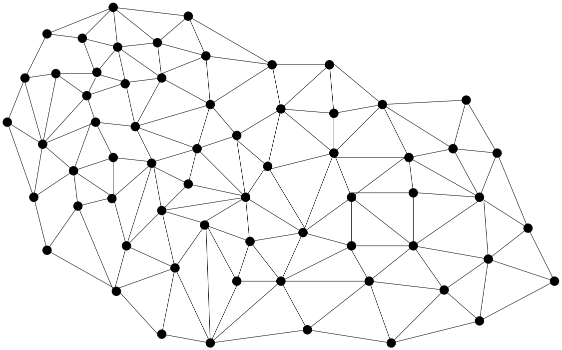
Route Guidance: Introduction

- given: sources s_k, targets t_k, demand rates d_k for traffic demands in a road network



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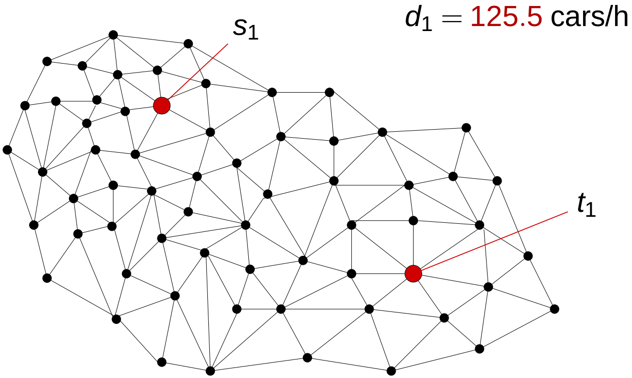

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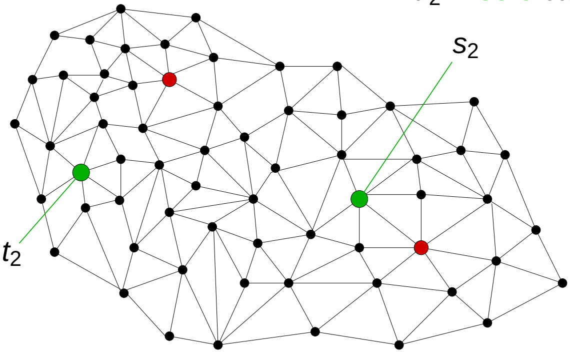

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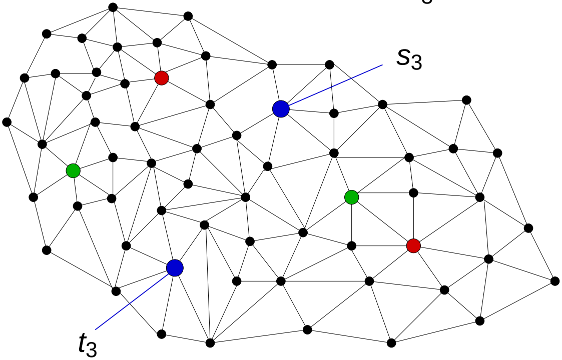

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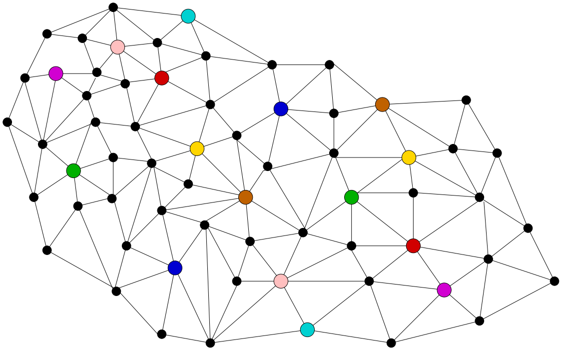

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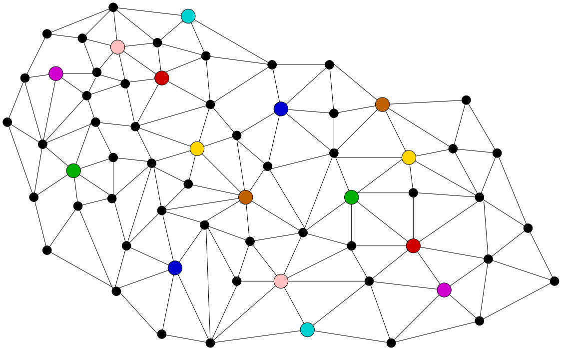

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Route Guidance: State of Technology

Route Guidance Systems...

... play an increasingly important role in today's traffic:

- in-car navigation systems
- urban road pricing schemes / centralized traffic routing

Today's systems use static data only:

- average travel times on road links
- locations / times of typical rush hour congestions
- locations of work zones

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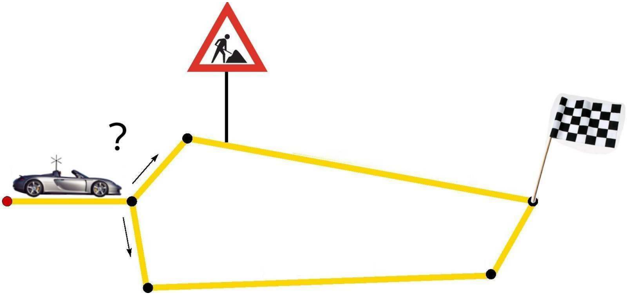
Results of Widespread Static Route Guidance

Travelers with the same origin and destination receive the same route suggestions:



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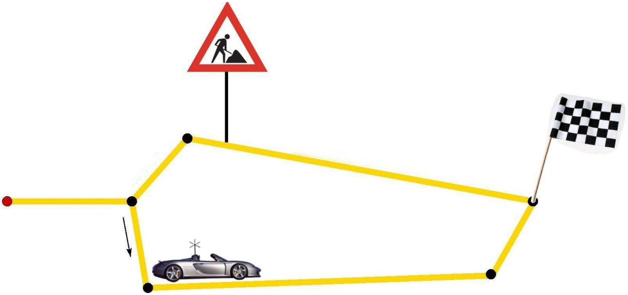

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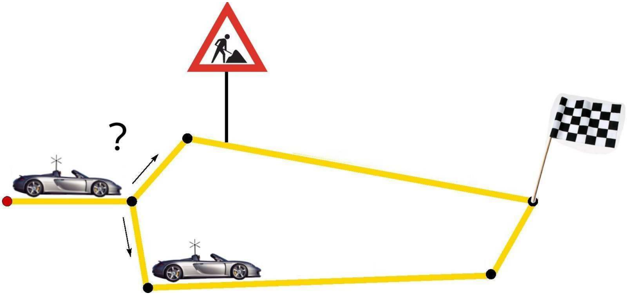

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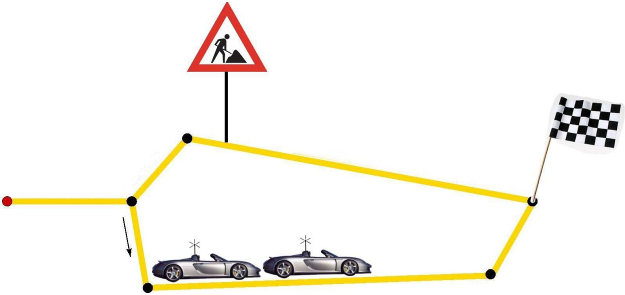

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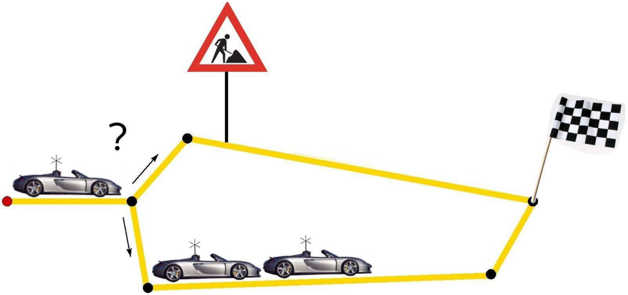

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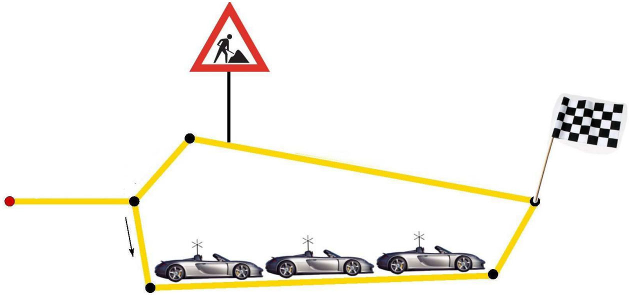

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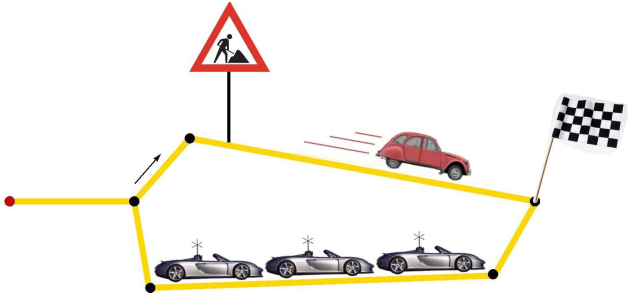

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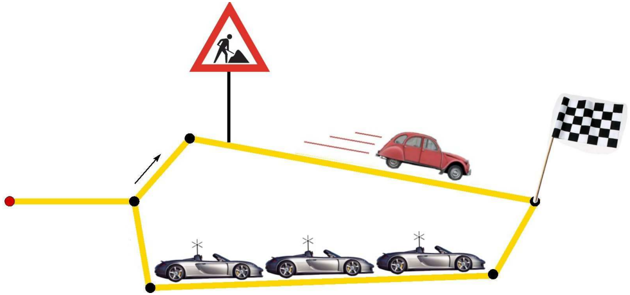

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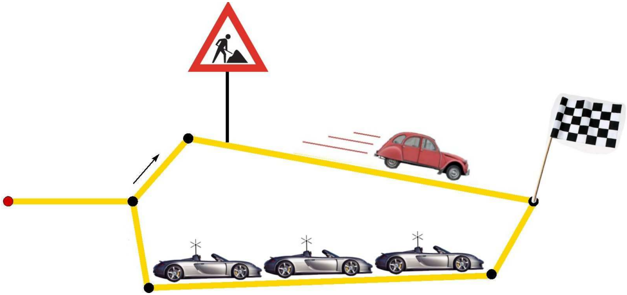

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The Need for Intelligent Traffic Routing

Problem

In order for Route Guidance Systems to help manage tomorrow's ever-increasing traffic demands, they must be able to evaluate travel times realistically.

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System Optimum

Sum of all travel times is minimal.

Problems (e.g. [Mahmassani and Peeta 1993]):

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No user can improve his travel time by individually changing his route.

$\Rightarrow$ "natural" flow pattern of unguided traffic

Result:

- "fair": drivers with same origin and destination have same travel times

Problems:

- sum of all travel times possibly a multiple of the one in system optimum ("price of anarchy", e.g. [Roughgarden and Tardos 2002])
- no indication about network performance (Braess paradox)

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System Optimum with Fairness Constraints

Idea [Jahn, Möhring, Schulz, Stier-Moses 2005]

Minimize sum of all travel times, but restrict usage of paths drivers would not accept:

- $\tau_p :=$ travel time on path p in UE
- $T_k :=$ travel time on paths chosen by commodity k in UE

$\Rightarrow$ only use paths p with

$$\tau_p \leq \varphi \cdot T_k$$

- suggestion: $\varphi = 1.02$

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Properties of the Constrained System Optimum

Results [Jahn, Möhring, Schulz, Stier-Moses 2005]

With appropriate φ, τ, solutions to CSO yield

- a lot more fairness than System Optimum
  - travel time of 99% of all users at most 30% higher than on fastest route.
  - in SO: 50%
- much better system performance than User Equilibrium
  - total travel time only $\frac{1}{3}$ as far away from SO as UE
- better routes for most drivers
  - 75% spend less travel time than in UE
  - only 0.4% spend 10% more (SO: 5%)

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The CSO Problem

min-cost multi-commodity flow problem with convex objective function and path constraints:

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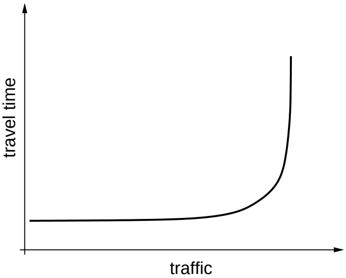
- CSO is non-linear: travel times vary with flow rate



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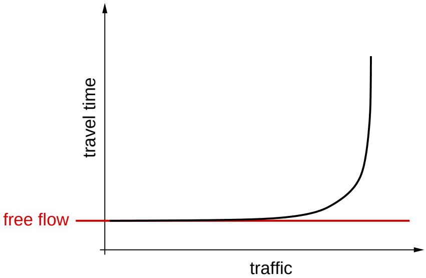

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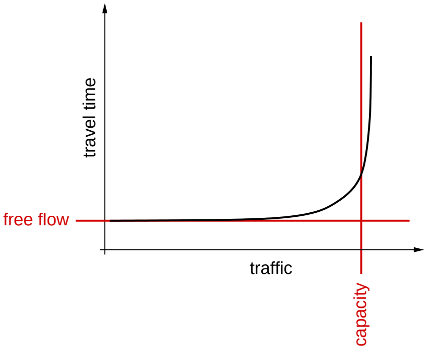

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- exponentially many paths in G
  $\Rightarrow$ cannot deal with variables x_p explicitly

Previous work [Jahn, Möhring, Schulz, Stier-Moses 2004]:

- solve CSO by variant of Frank-Wolfe convex combinations algorithm and constrained shortest path calculations

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- use cutting plane method to solve dual problem

- similar approach successfully applied to other multi-commodity flow problems [Babonneau and Vial 2005]

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& \sum_{p\in P_k} x_p = d_k & k \in K \\
& \tau_p \leq \varphi T_k & p \in P^k : x_p > 0;\ k \in K \\
& x_p \geq 0 & p \in P
\end{array}
$$

Lagrangian Relaxation for CSO

- add penalty terms with multipliers u_j to objective

$$
\begin{array}{lll}
\textit{Minimize} & L(x,u) := \sum_{a\in A} l_a(x_a)x_a + \sum_{a\in A}\left(u_a \cdot \left(\sum_{k\in K} z_a^k - x_a\right)\right) & \\
\textit{subject to} & \sum_{p\in P_k : a\in p} x_p = z_a^k & a \in A \\
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\end{array}
$$

Lagrangian Relaxation for CSO

- remaining constraints resemble those of
  $|K|$ constrained shortest path problems in z_a^k

$$
\begin{array}{lll}
\textit{Minimize} & L(x,u) := \sum_{a\in A} l_a(x_a)x_a + \sum_{a\in A} \left(u_a \cdot \left(\sum_{k\in K} z_a^k - x_a \right) \right) & \\
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Lagrangian Relaxation for CSO

- Lagrangian separable in x and z?

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Lagrangian Relaxation for CSO

- Lagrangian separable in x and z?

$$\text{Minimize} \quad L(x,u) := \overbrace{\sum_{a\in A} (l_a(x_a) - u_a)\cdot x_a}^{L_1(x,u)} + \overbrace{\sum_{k\in K}\sum_{a\in A} u_a \cdot z_a^k}^{L_2(z,u)}$$

$$\text{subject to} \quad \sum_{p\in P_k : a\in p} x_p = z_a^k \qquad a\in A$$

$$\sum_{p\in P_k} x_p = d_k \qquad k\in K$$

$$\tau_p \leq \varphi T_k \qquad p\in P^k : x_p > 0;\ k\in K$$

$$x_p \geq 0 \qquad p\in P$$

Lagrangian Relaxation for CSO

⇝ Yes!

$$
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$$

Lagrangian Relaxation for CSO

- easier problem: analytical minimization in x...

$$\textit{Minimize} \quad L(x,u) := \overbrace{\sum_{a\in A} (l_a(x_a) - u_a)\cdot x_a}^{L_1(x,u)} + \overbrace{\sum_{k\in K}\sum_{a\in A} u_a \cdot z_a^k}^{L_2(z,u)}$$

$$\begin{array}{llll} \textit{subject to} & \sum_{p\in P_k : a\in p} x_p = z_a^k & & a \in A \\ & \sum_{p\in P_k} x_p = d_k & & k \in K \\ & \tau_p \le \varphi T_k & & p \in P^k : x_p > 0;\ k \in K \\ & x_p \ge 0 & & p \in P \end{array}$$

Lagrangian Relaxation for CSO

- ...and $|K|$ constrained shortest path problems in z^k

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$$

Lagrangian Relaxation for CSO

- up next: dual problem (maximize this minimum over u)

$$\text{Minimize} \quad L(x,u) := \overbrace{\sum_{a\in A} (l_a(x_a) - u_a)\cdot x_a}^{L_1(x,u)} + \overbrace{\sum_{k\in K}\sum_{a\in A} u_a \cdot z_a^k}^{L_2(z,u)}$$

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Analytic Center Cutting Plane Method

- approximation scheme for maximization of a concave function over a convex set
- implementation by Babonneau, Vial et. al. at LogiLab, University of Geneva

Two components:

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  - manages a localization set containing all optimal points
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  - generates cutting planes to further bound the localization set
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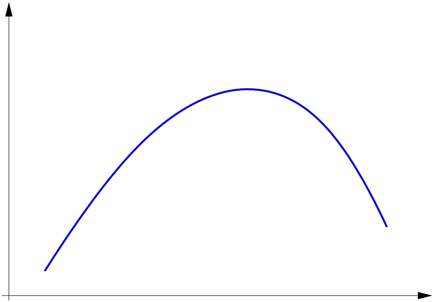
Oracle for CSO

- evaluate objective function ⇝ CSP calculations
- calculate subgradient at query point ⇝ easy

⇒ subgradients and best objective value define cutting planes bounding the localization set



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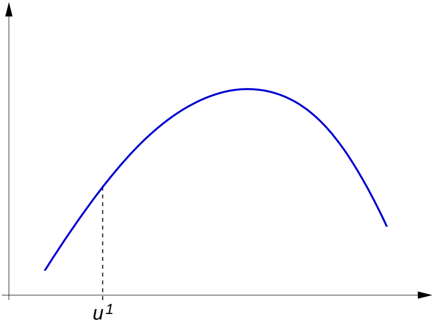

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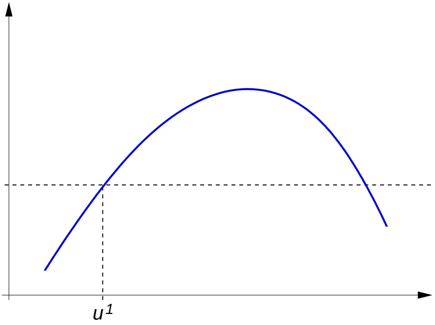

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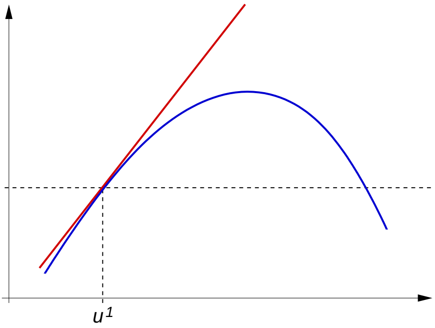

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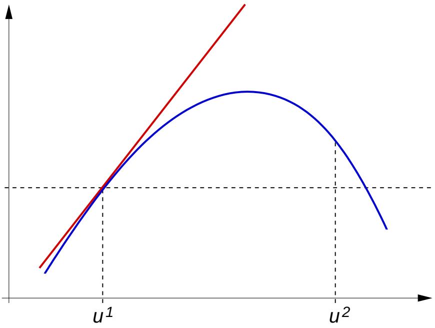

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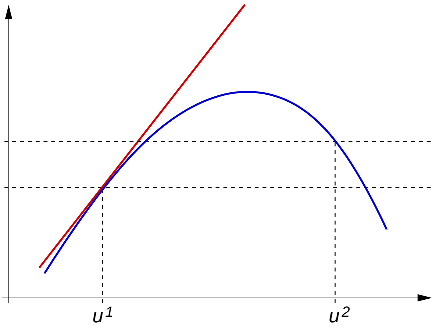

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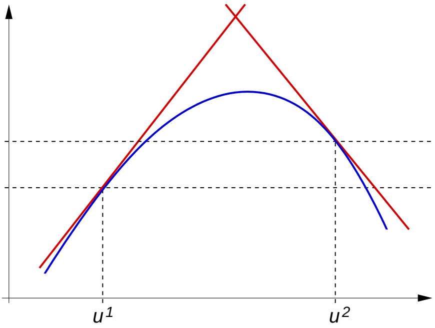

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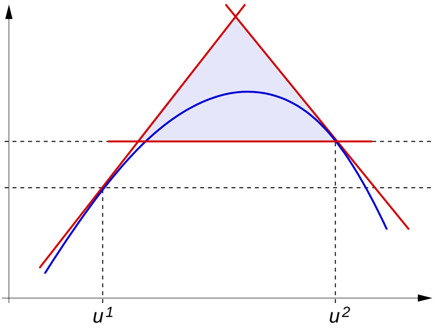
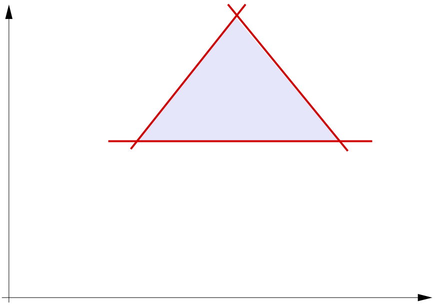


Query Points

- analytic center: maximum distances from cutting planes
  - calculation by damped Newton method
- u-component is next query point



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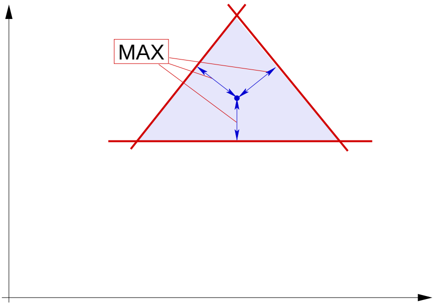

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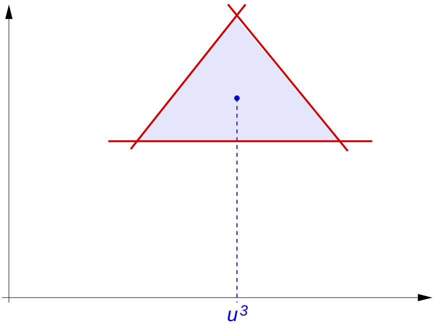

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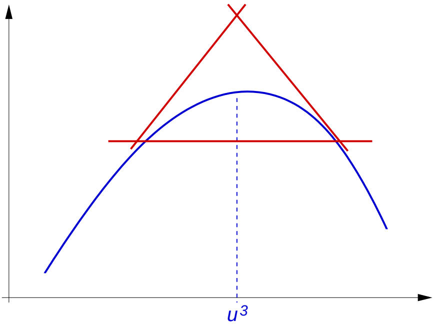
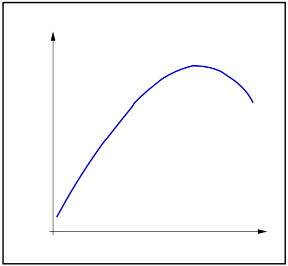
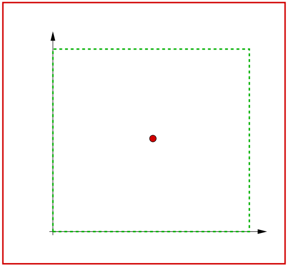


Illustration of an ACCPM Run



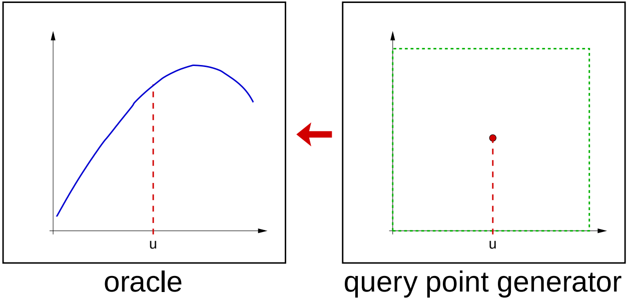
oracle



query point generator

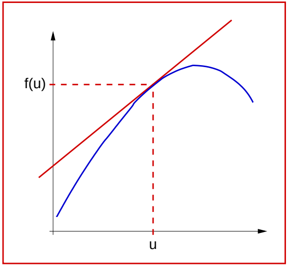
- localization set artificially bounded $\Rightarrow$ compact

Illustration of an ACCPM Run



In each iteration, a query point is sent to the oracle,...

Illustration of an ACCPM Run



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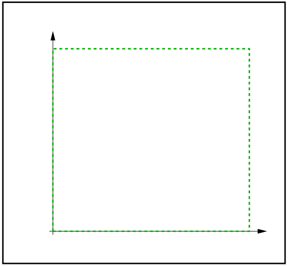
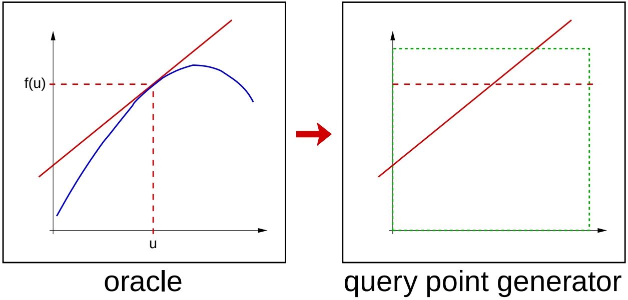


query point generator

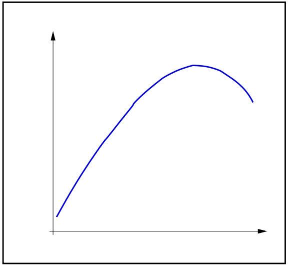
... the value and subgradient of θ are calculated...

Illustration of an ACCPM Run



... which define cutting planes...

Illustration of an ACCPM Run



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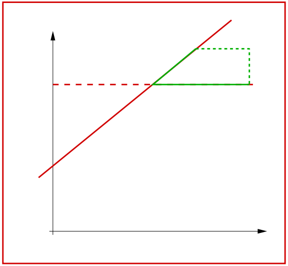
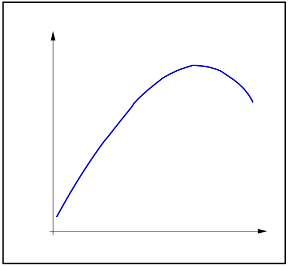


query point generator

... to further bound the localization set.

Illustration of an ACCPM Run



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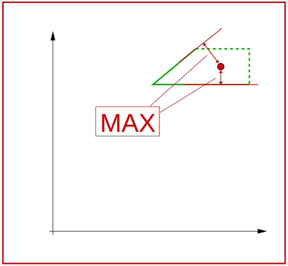
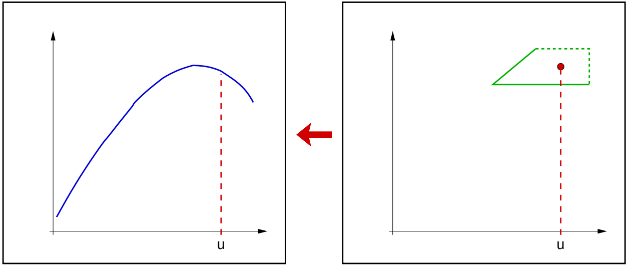


query point generator

Then, the proximal analytic center is calculated...

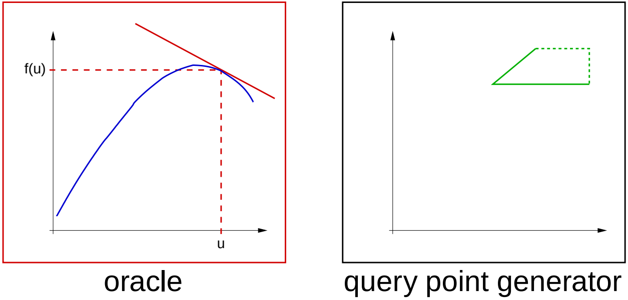
Illustration of an ACCPM Run



oracle query point generator

... which defines the next query point.

Illustration of an ACCPM Run



Process is repeated...

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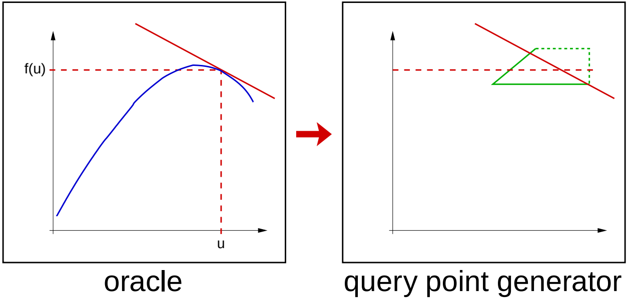

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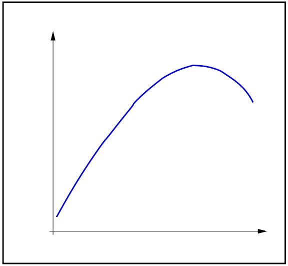

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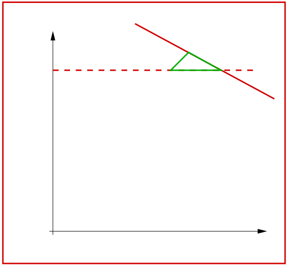

query point generator

Process is repeated...

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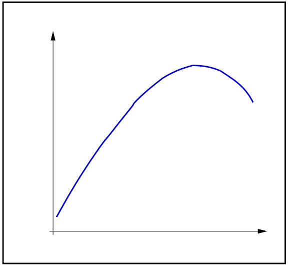

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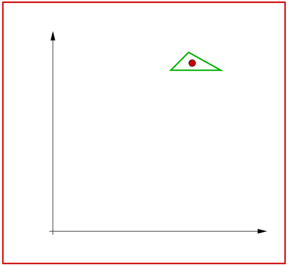

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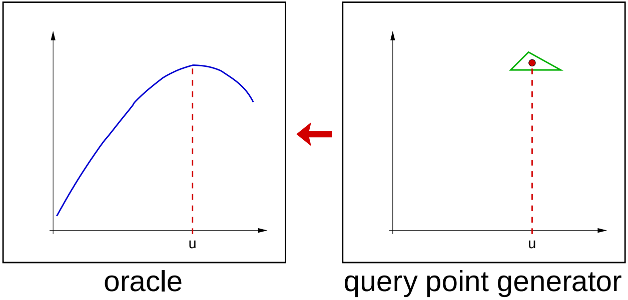

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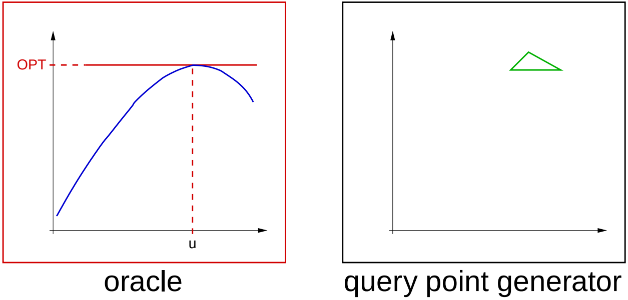

... until desired precision is achieved.

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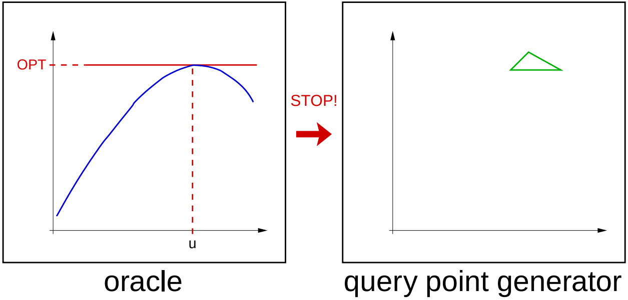

... until desired precision is achieved.

There is much more inside ACCPM

Accelerating convergence:

- sophisticated parameters for dynamic weighting of cuts
- cut elimination techniques
- rules for updating the proximal reference point

Problem specific functionalities:

- multiple cuts per iteration
- active set strategies

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Test Instances

Name	$\|V\|$	$\|A\|$	$\|K\|$
Sioux Falls	24	76	528
Winnipeg	1052	2836	4344
Neukoelln	1890	4040	3166
Chicago Sketch	933	2950	83113

- all but `Neukoelln` from Transportation Network Problems online database
- tested on Intel Pentium 4 2.8GHz with 1 GB RAM, SuSE Linux
- optimality gap: 0.5%

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- all but Neukoelln from Transportation Network Problems online database
- tested on Intel Pentium 4 2.8GHz with 1 GB RAM, SuSE Linux
- optimality gap: 0.5%

Test Instances

Name	$\|V\|$	$\|A\|$	$\|K\|$
`Sioux Falls`	24	76	528
`Winnipeg`	1052	2836	4344
`Neukoelln`	1890	4040	3166
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General Findings

- runtime of query point generator negligible
  - almost all calculation time spent finding CSPs
- goal-directed approach slowest for free flow travel times
  - time for preprocessing longer than resulting speed-up

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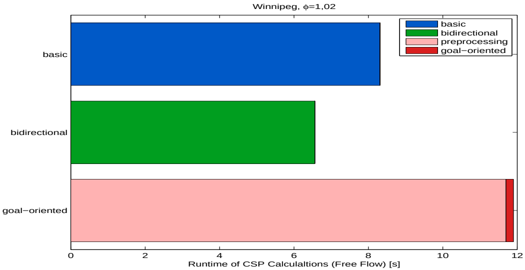

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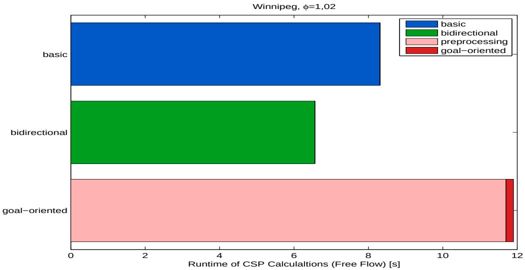
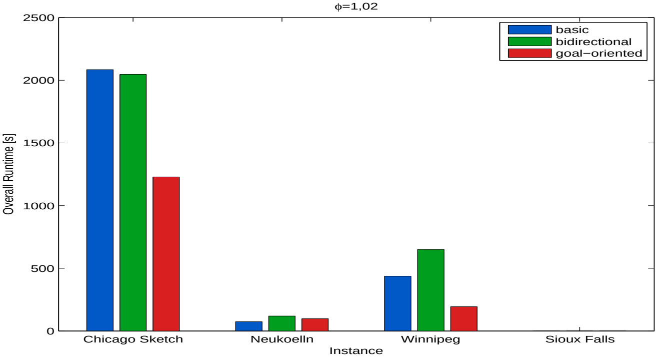


Overall Runtime

- total runtime: basic / bidirectional / goal-oriented



- why does goal-oriented algorithm perform best?

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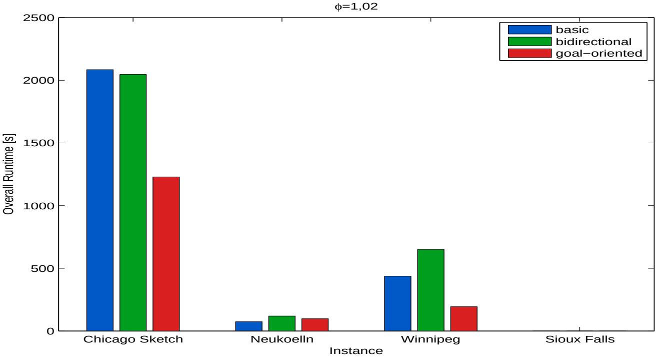

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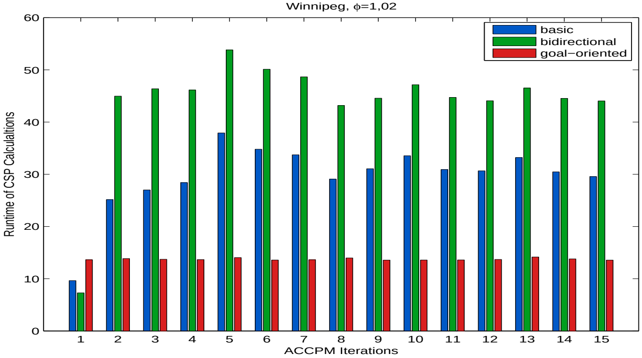
Effect of CSP Acceleration

- CSP-runtime over ACCPM iterations for Winnipeg



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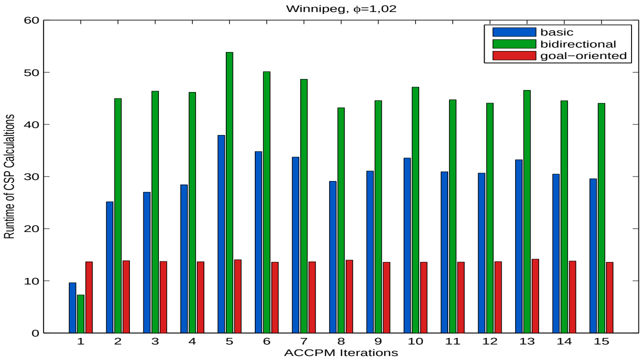

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Explanation

- edge length for CSP calculations: dual variables u
- as we approach optimum, u approaches CSO travel times

$\Rightarrow$ in congested networks, direct paths become unattractive
$\Rightarrow$ basic labeling algorithm is deflected from target
  $\rightsquigarrow$ infeasible paths are explored first

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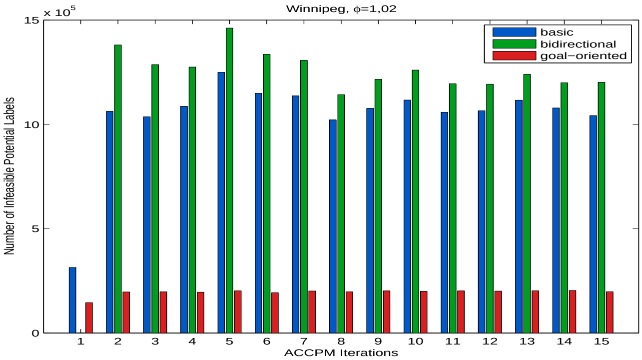
Exploration of Nodes on Infeasible Paths

- potential labels violating length bounds for Winnipeg



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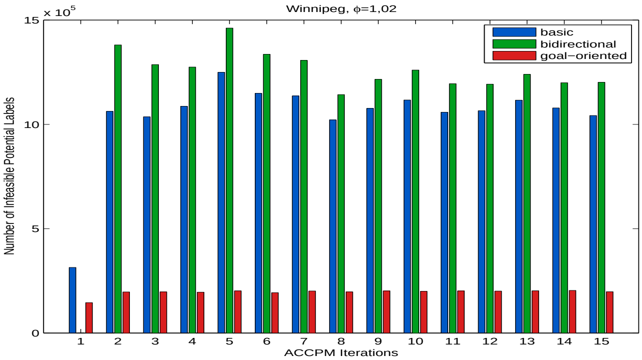

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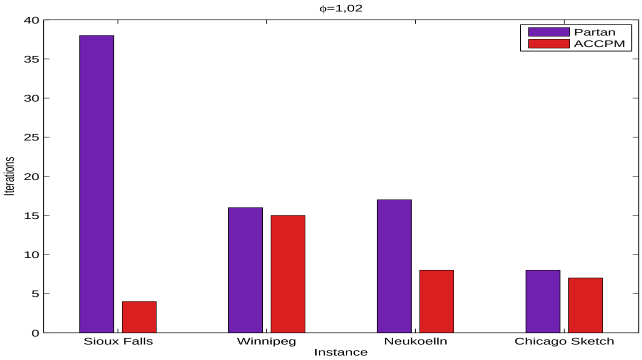
Comparison with Partan

- number of iterations compared with Partan



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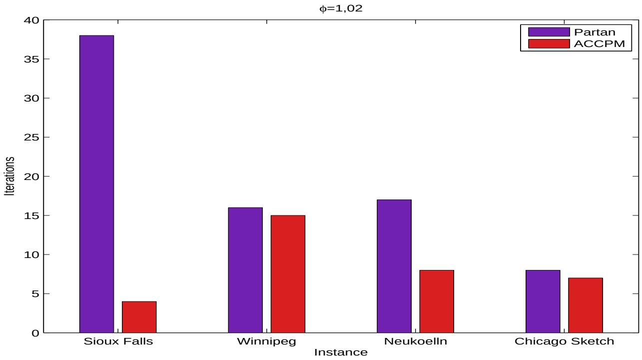

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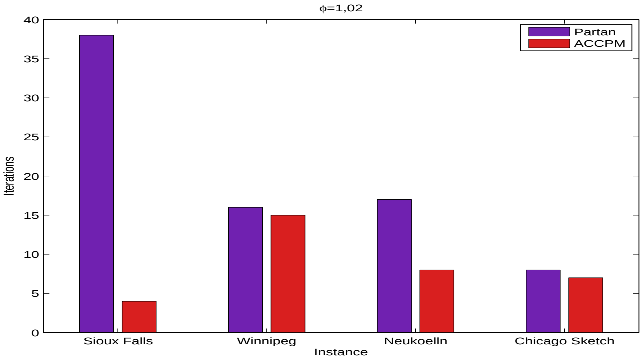

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Outline

1. Traffic Flow Optimization under Fairness Constraints
   - Motivation
   - The Constrained System Optimum Problem (CSO)
2. Solving the CSO Problem
   - Lagrangian Relaxation to Treat Non-Linearity
   - Proximal-ACCPM: An Interior Point Cutting Plane Method
3. Results
   - Computational Study
   - Summary

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- constrained system optimum delivers equally good and fair solutions for traffic optimization
- proximal-ACCPM can be used to solve a Lagrangian relaxation of CSO
- algorithm outperforms previous approaches: $\approx \frac{1}{2}$ the runtime of Partan algorithm
- interesting relationship

$$\begin{aligned} & \text{dual variables} \\ \leftrightarrow\ & \text{level of congestion} \\ \leftrightarrow\ & \text{runtime of different CSP algorithms} \end{aligned}$$

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Thank you for your attention!

Questions?