Preserving Confidentiality AND Providing Adequate Data for Statistical Modeling

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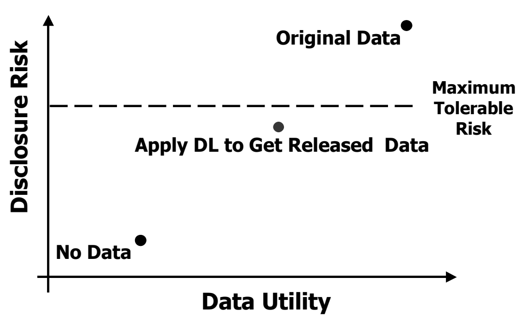
Overview

- **Background and some fundamental abstractions for disclosure limitation.**
  - **Statistical users want more than to retrieve a few numbers.**
- **Results on bounds for table entries.**
- **Uses of Markov bases for exact distributions and perturbation of tables.**
- **Links to log-linear models, and related statistical theory and methods.**

R-U Confidentiality Map



(Duncan, et al. 2001)

NISS Prototype Query System

- **For *k*-way table of counts.**
- ***Queries:* Requests for marginal tables.**
- ***Responses:* Yes--release; No; (and perhaps "Simulate" and then release).**
- **As released margins cumulate we have increased information about table entries.**
- **Margins need to be consistent ==> possible simulated releases get highly constrained.**

Confidentiality Concern

- Uniqueness in population table ⇔ cell count of "1".
- Uniqueness allows intruder to match characteristics in table with other data bases that include the same variables plus others to learn confidential information.
  - Assuming data are reported without error!
- Identity versus attribute disclosure.

5

Fundamental Abstractions

- Query space, Q, with partial ordering:
  - Elements can be marginal tables, conditionals, *k*-groupings, regressions, or other data summaries.
  - *Released set:* R(*t*), and implied *Unreleasable set:* U(*t*).
  - *Releasable frontier:* maximal elements of R(*t*).
  - *Unreleasable frontier:* minimal elements of U(*t*).
- *Risk* and *Utility* defined on subsets of Q.
  - *Risk Measure*: identifiability of small cell counts.
  - *Utility*: reconstructing table using log-linear models.
  - Release rules must balance risk and utility:
    - R-U Confidentiality map.
    - General Bayesian decision-theoretic approach.

6

Why Marginals?

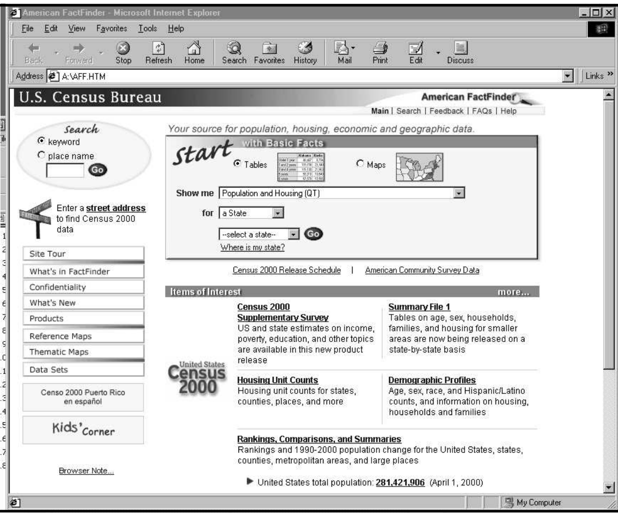
- Simple summaries corresponding to subsets of variables.
- Traditional mode of reporting for statistical agencies and others.
- Useful in statistical modeling: Role of log-linear models.
- Collapsing categories of categorical variables uses similar DL methods and statistical theory.

7

Example 1: 2000 Census

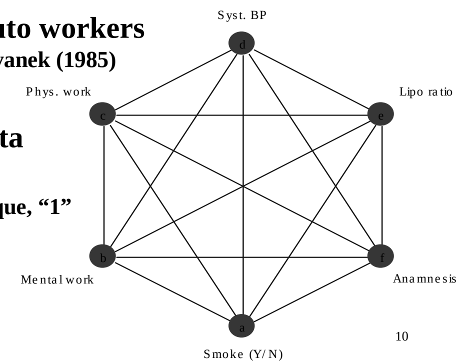
- U.S. decennial census "long form"
  - 1 in 6 sample of households nationwide.
  - 53 questions, many with multiple categories.
  - Data measured with substantial error!
  - Data reported after application of data swapping!
- Geography
  - 50 states; 3,000 counties; 4 million "blocks".
  - Release of detailed geography yields uniqueness in sample and at some level in population.
- *American Factfinder* releases various 3-way tables at different levels of geography.

8



Example 2: Risk Factors for Coronary Heart Disease

- **1841 Czech auto workers**
  Edwards and Havanek (1985)
- **2^6 table**
- **population data**
  - **"0" cell**
  - **population unique, "1"**
  - **2 cells with "2"**



Example 2: The Data

F	E	D	C	B: no, A: no	B: no, A: yes	B: yes, A: no	B: yes, A: yes
neg	< 3	< 140	no	44	40	112	67
			yes	129	145	12	23
		≥ 140	no	35	12	80	33
			yes	109	67	7	9
	≥ 3	< 140	no	23	32	70	66
			yes	50	80	7	13
		≥ 140	no	24	25	73	57
			yes	51	63	7	16
pos	< 3	< 140	no	5	7	21	9
			yes	9	17	1	4
		≥ 140	no	4	3	11	8
			yes	14	17	5	2
	≥ 3	< 140	no	7	3	14	14
			yes	9	16	2	3
		≥ 140	no	4	0	13	11
			yes	5	14	4	4

Example 3: NLTCS

- **National Long Term Care Survey**
  - **20-40 demographic/background items.**
  - **30-50 items on disability status, ADLs and IADLs, most binary but some polytomous.**
  - **Linked Medicare files.**
  - **5 waves: 1982, 1984, 1989, 1994, 1999.**
- **We've been working with 2^{16} table, collapsed across several waves of survey, with *n*=21,574.**

  Erosheva (2002)

  Dobra, Erosheva, & Fienberg(2003)

Two-Way Fréchet Bounds

- For 2×2 tables of counts $\{n_{ij}\}$ given the marginal totals $\{n_{1+},n_{2+}\}$ and $\{n_{+1},n_{+2}\}$:

n_{11}	n_{12}	n_{1+}
n_{21}	n_{22}	n_{2+}
n_{+1}	n_{+2}	n

$$\min(n_{i+}, n_{+j}) \geq n_{ij} \geq \max(n_{i+} + n_{+j} - n, 0)$$

- Interested in multi-way generalizations involving higher-order, overlapping margins.

Bounds for Multi-Way Tables

- *k*-way table of non-negative counts, $k \geq 3$.
  - Release set of marginal totals, possibly overlapping.
  - *Goal*: Compute bounds for cell entries.
  - LP and IP approaches are NP-hard.
- Our strategy has been to:
  - Develop efficient methods for several special cases.
  - Exploit linkage to statistical theory where possible.
  - Use general, less efficient methods for residual cases.
- Direct generalizations to tables with non-integer, non-negative entries.

Role of Log-linear Models?

- For 2×2 case, lower bound is evocative of MLE for estimated expected value under independence:

$$\hat{m}_{ij} = n_{i+}n_{+j} / n.$$

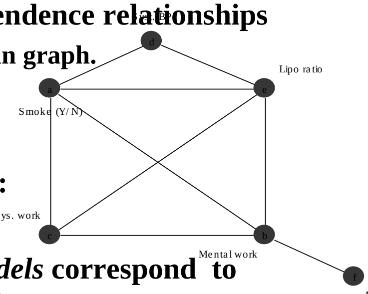
  - Bounds correspond to log-linearized version.
  - Margins are *minimal sufficient statistics (MSS)*.
- In *3*-way table of counts, $\{n_{ijk}\}$, we model logs of expectations $\{E(n_{ijk})=m_{ijk}\}$:

$$\log(m_{ijk}) = u + u_{1(i)} + u_{2(j)} + u_{3(k)} + u_{12(ij)} + u_{13(ik)} + u_{23(jk)}$$

- *MSS* are margins corresponding to highest order terms: $\{n_{ij+}\}$, $\{n_{i+k}\}$, $\{n_{+jk}\}$.

Graphical & Decomposable Log-linear Models

- *Graphical models:* defined by simultaneous conditional independence relationships
  - Absence of edges in graph.



Example 2:

Czech autoworkers

Graph has 3 cliques:

[ADE][ABCE][BF]

- *Decomposable models* correspond to triangulated graphs.

MLEs for Decomposable Log-linear Models

- **For decomposable models, expected cell values are explicit function of margins, corresponding to MSSs (*cliques* in graph):**
  - **For conditional independence in 3-way table:**

$$\log m_{ijk} = u + u_{1(i)} + u_{2(j)} + u_{3(k)} + u_{12(ij)} + u_{13(ik)}$$

$$m_{ijk} = \frac{m_{ij+} m_{i+k}}{m_{i++}}$$

- **Substitute observed margins for expected in explicit formula to get MLEs.**

17

Multi-way Bounds

- **For decomposable log-linear models:**

$$\text{Expected Value} = \frac{\prod MSSs}{\prod Separators}$$

- ***Theorem:* When released margins correspond to those of a decomposable model:**
  - ***Upper bound:* minimum of relevant margins.**
  - ***Lower bound:* maximum of zero, or sum of relevant margins minus separators.**
  - **Bounds are sharp.**

Fienberg and Dobra (2000)

18

Multi-Way Bounds (cont.)

- ***Example*: Given margins in *k*-way table that correspond to (*k*-1)-fold conditional independence given variable 1:**

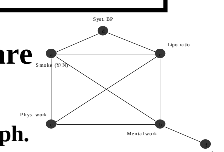
$$\{n_{i_1 i_2 + \ldots +}\}\ \{n_{i_1 + i_3 \ldots +}\} \ldots . \{n_{i_1 + \ldots + i_k}\}$$

- **Then bounds are**

$$\min\{n_{i_1 i_2 + \ldots ++}, n_{i_1 + i_3 \ldots ++}, \ldots, n_{i_1 + \ldots + i_k}\} \geq n_{i_1 i_2 i_3 \ldots i_k}$$
$$\geq \max\{n_{i_1 i_2 + \ldots ++} + n_{i_1 + i_3 \ldots ++} + \ldots + n_{i_1 + \ldots + i_k} - n_{i_3 ++ \ldots ++}(k-2), 0\}$$

19

Ex. 2: Czech Autoworkers



- **Suppose released margins are [ADE][ABCE][BF] :**
  - **Correspond to decomposable graph.**
  - **Cell containing population unique has bounds [0, 25].**
  - **Cells with entry of "2" have bounds: [0,20] and [0,38].**
  - **Lower bounds are all "0".**
- **"Safe" to release these margins; low risk of disclosure.**

20

Bounds for [BF][ABCE][ADE]

			B	no		yes	
F	E	D	C / A	no	yes	no	yes
neg	< 3	< 140	no	[0,88]	[0,62]	[0,224]	[0,117]
			yes	[0,261]	[0,246]	[0,25]	[0,38]
		≥ 140	no	[0,88]	[0,62]	[0,224]	[0,117]
			yes	[0,261]	[0,151]	[0,25]	[0,38]
	≥ 3	< 140	no	[0,58]	[0,60]	[0,170]	[0,148]
			yes	[0,115]	[0,173]	[0,20]	[0,36]
		≥ 140	no	[0,58]	[0,60]	[0,170]	[0,148]
			yes	[0,115]	[0,173]	[0,20]	[0,36]
pos	< 3	< 140	no	[0,88]	[0,62]	[0,126]	[0,117]
			yes	[0,134]	[0,134]	[0,25]	[0,38]
		≥ 140	no	[0,88]	[0,62]	[0,126]	[0,117]
			yes	[0,134]	[0,134]	[0,25]	[0,38]
	≥ 3	< 140	no	[0,58]	[0,60]	[0,126]	[0,126]
			yes	[0,115]	[0,134]	[0,20]	[0,36]
		≥ 140	no	[0,58]	[0,60]	[0,126]	[0,126]
			yes	[0,115]	[0,134]	[0,20]	[0,36]

Table 1 - Bounds for Autoworkers data given the marginals [BF], [ABCE], [ADE].

Example 2 (cont.)

- **Among all 32,000+ decomposable models, the tightest possible bounds for three target cells are: (0,3), (0,6), (0,3).**
  - **31 models with these bounds! All involve [ACDEF].**
  - **Another 30 models have bounds that differ by 5 or less (*critical width*) and these involve [ABCDE].**
  - **Method used to search for "optimal" decomposable release also identifies [ABDEF] as potentially problematic.**
- **Allows proper statistical test of fit for most interesting models.**

More on Bounds

- **Extension for log-linear models and margins corresponding to reducible graphs.**
- **For 2^k tables with (*k*-1) dimensional margins fixed (need one extra bound here and it comes from log-linear model theory: existence of MLEs).**
  - **Extend to general *k*-way case by looking at all possible collapsed 2^k tables.**
- **General "shuttle" algorithm in Dobra (2002) works for all cases.**
  - **Also generates most special cases with limited extra computation.**

Example 2: Release of All 5-way Margins

- **Approach for 2×2×2 generalizes to 2^k table given (k-1)-way margins.**
- **In 2^6 table, if we release all 5-way margins:**
  - **Almost identical upper and lower values; they all differ by 1.**
  - **Only 2 feasible tables with these margins!**
- **UNSAFE!**

Example 3: NLTCS

- 2^{16} table of ADL/IADLs with 65,536 cells:
  - 62,384 zero entries; 1,729 cells with count of "1" and 499 cells with count of "2".
  - n=21,574.
  - Largest cell count: 3,853---no disabilities.
- Used simulated annealing algorithm to search all decomposable models for "decomposable" model on frontier with max[upper bound – lower bound] >3.
- Acting *as if* these were *population* data.

NLTCS Search Results

- Decomposable frontier model:

  {[1,2,3,4,5,7,12], [1,2,3,6,7,12], [2,3,4,5,7,8], [1,2,4,5,7,11], [2,3,4,5,7,13], [3,4,5,7,9,13], [2,3,4,5,13,14], [2,4,5,10,13,14], [1,2,3,4,5,15], [2,3,4,5,8,16]}.

- Has one 7-way and eight 6-way marginals.

Perturbation Maintaining Marginal Totals

	w_1	w_2	w_3	w_4
v_1	+1	0	–1	0
v_2	–1	0	+1	0
v_3	0	0	0	0
v_4	0	0	0	0

- Perturbation distributions given marginals require Markov basis for perturbation moves.

Perturbation for Protection

- Perturbation preserving marginals involves a parallel set of results to those for bounds:
  - Markov basis elements for decomposable case requires only "simple" moves. (Dobra, 2002)
  - Efficient generation of Markov basis for reducible case. (Dobra and Sullivent, 2002)
  - Simplifications for 2^k tables ("binomials").
  - Rooted in ideas from likelihood theory for log-linear models and computational algebra of toric ideals.

Some Ongoing Research

- Queries in form of combinations of marginals and conditionals.
- Inferences from marginal releases.
- What information does the intruder really have?
- Record linkage and matching.
- Simplified cyclic perturbation distributions.
- Computational algebraic statistics.

29

Summary

- Some fundamental abstractions for disclosure limitation.
- Results on bounds for table entries.
- Parallels for Markov bases for exact distributions and perturbation of tables.
- New theoretical links among disclosure limitation, statistical theory, and computational algebraic geometry.

30

The End

- Most papers available for downloading at

http://www.niss.org

http://www.stat.cmu.edu/~fienberg/disclosure.html

- Workshop on Computational Algebraic Statistics December 14 to 18, 2003, American Institute of Mathematics, Palo Alto, California

http://aimath.org/ARCC/workshops/compalgstat.html

31

Stochastic Perturbation Methods

- Some methods well-developed in statistical literature:
  - Matrix masking, including adding noise
  - Post-randomization
    - Randomized response after data are collected
  - Multiple Imputation
    - Sampling from full posterior distribution
  - Data swapping and constrained cyclic perturbation
- Key is full information on stochastic transformation for proper statistical inferences.

32

Exact Distribution of Table Given Marginals

- **Exact probability distribution for log-linear model given its MSS marginals:**

$$\sigma(\mathrm{n}) = \frac{\prod_{i \in I} \frac{1}{n(i)!}}{\sum_{m \in S(c)} \left(\prod_{i \in I} \frac{1}{m(i)!} \right)}$$

  - **Can generate distribution using Diaconis-Sturmfels (1998) MCMC approach using Markov basis.**

Fienberg, Makov, Meyer, Steele (2002)

33

Markov Basis "Moves"

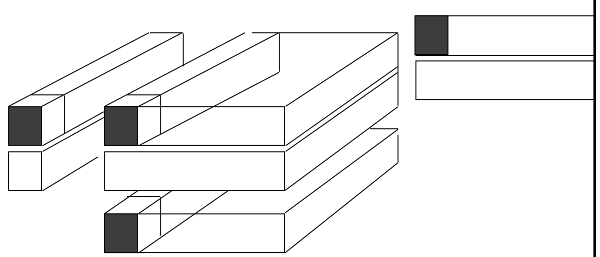
- **Simple moves:**
  - **Based on standard linear contrasts involving 1's, 0's, and -1's for embedded 2^l subtables.**
  - **For example, in 2×2×2 table, there is 1 move of form:**

$$\begin{matrix} 1 & -1 \\ -1 & 1 \end{matrix} \qquad \begin{matrix} -1 & 1 \\ 1 & -1 \end{matrix}$$

- **"Non-simple" moves:**
  - **Require combination of simple moves to reach extremal tables in convex polytope.**

34

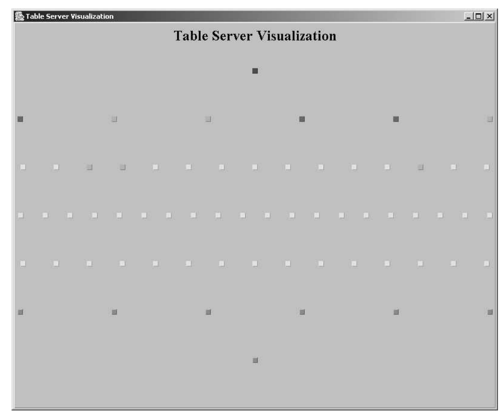
Three-way Illustration (k=3)



Challenge: **Scaling up approach for large *k*.**

35

NISS Table Server: 6-Way Table



36