Secure Computation (a tutorial)

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Cryptology - The First Few Millennia

Well Done!
Thank you, Sir Cryptographer!

Curses! I cannot read the message!

Goal of cryptology - protect messages from prying eyes.
Lockboxes for data: data safe as long as it is locked up.

The Last Twenty Years

Then: data protected, but not used.

Now: Use data, but still protect it as much as possible.

Secure Computation:
Can we combine information while protecting it as much as possible?

The Love Game (AKA the AND game)

He loves me, he loves me not...

She loves me, she loves me not...

Want to know if both parties are interested in each other.
But... Do not want to reveal unrequited love.

Input = 1: I love you
Input = 0: I love you... as a friend
Must compute F(X,Y)=X\land Y, giving F(X,Y) to both players.
Can we reveal the answer without revealing the inputs?
The Spoiled Children Problem
(aka The Millionaires Problem [Yao])

Who has more toys?
Who Cares?

Pearl wants to know whether she has more toys than Gersh. Doesn't want to tell Gersh anything.

Gersh is willing for Pearl to find out who has more toys, Doesn't want Pearl to know how many toys he has.

Can we give Pearl the information she wants, and nothing else, without giving Gersh any information at all?

Auctions with Private Bids

Normal auction: Players reveal bids - high bid is identified
 private sealed bid system, but kept private

Only the winning bid, bidders are revealed.

Drawback: Revealing the losing bids gives away strategic information that bidders and auctioneers might exploit in later auctions.

Can we have private bids where no one, not even the auctioneer, knows the losing bids?

Electronic Voting

Final tally:
War: 2
Peace: 2
Nader: 1

The winner is: War

Secure Computation
(Yao, Goldreich-Micali-Wigderson)

Players: 1,...,N
Inputs: X1,...,Xn
Outputs: F1(X1,...,Xn),...,Fn(X1,...,Xn)
Players should learn correct outputs and nothing else.
AA Ekekeff Protocol

I'll Help!
(for a reasonable consulting fee...)

F₁(X₁, X₂)

F₂(X₁, X₂)

X₁

X₂

Goal: Implement something that "looks like" ideal protocol.

The Nature of the Enemy

Corrupting a player lets adversary:
Learn its input/output
See everything it knew, saw, later sees.
Control its behavior (e.g., messages sent)

What can go wrong?

War

War

War

War

Peace

Final Tally: Red-Blooded-American Patriots: $ Terrorist-Sympathizing Liberals: $
The winner is: War-War

Privacy: Inputs should not be revealed.
Correctness: Answer should correspond to inputs.

What We Can/Can't Hope For

Corrupted players have no privacy on inputs/outputs.
Outputs may reveal inputs:
If candidate received 100% of the votes, we know how you voted.
Cannot complain about adversary learning what it can by (independently) selecting its inputs and looking at its outputs.
Cannot complain about adversary altering outcome solely by (independently) altering its inputs.
Goal is to not allow the adversary to do anything else.
Definitions very subtle: Beaver, Micali-Rogaway, Canetti...
Can We Do It?

Yao (GMW,GV,K,...):
Yes (for two party case)*
Cryptographic solutions require "reasonable assumptions"
   e.g., hardness of factoring
'Slight issues about both players getting answer at same time.

Goldreich-Micali-Wigderson (BGW,CCD,RB,Bea,...):
Yes, if number of parties corrupted is less than some constant
fraction of the total number of players (e.g., √n/2, √n/3).
No hardness assumptions necessary.
As long as functions are computable in polynomial time,
solutions require polynomial computation, communication.

Can We Really Do It?

General solutions as impractical as they are beautiful.

Step 1:
   Break computations to be performed into itsy-bitsy steps.
   (additions, multiplications, bitwise operations)

Step 2:
   For each operation...

Step 3:
   Despair at how many itsy-bitsy steps your computation
   takes.
   Is there any hope?

Signs of Hope

Sometimes, don't need too many itsy-bitsy operations.

Naor-Pinkas-Sumner
   Functions computed when running auctions are simple.
   Highly optimize Yao-like constructions.

Testing if two strings are equal is very practical.

Can exploit algebraic structure to minimize work.
Rabin: Can compute sums very efficiently

Electronic Voting

Most extensively researched subarea of secure computation.

Protocols are now very practical.
   100,000 voters a piece of cake,
   1,000,000 voters doable.

Several commercial efforts
   Chaum, Neff, NEC,...

Many interesting issues, both human and technical:
   What should our definitions be?
Distributed Cryptographic Entities

Public Key: P
Secret Key: S

Trusted public servant cheerfully encrypts, decrypts, signs messages, when appropriate.

Blindly:
- Can break secret key up among several entities,
- Can still encrypt, decrypt, sign,
- Remains secure even if a few parties are corrupted.

And Sometimes There's Magic

Chor-Goldreich-Kushilevitz-Sudan,...Kushilevitz-Ostrovsky,...

Private information retrieval:

Can you download a data entry from a repository without letting the repository know what you're interested in?

Conclusions

Secure computation is an extremely powerful framework.

Very rich general theory.

A few applications now ready for prime time.

Keep watching this space!