

k-Anonymous Message Transmission

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Sender Anonymous Protocol
Adversary cannot identify the sender
of a particular message

Sender Anonymous Protocol
Adversary cannot identify the sender
of a particular message

Receiver Anonymous Protocol
Adversary cannot identify the receiver
of a particular message

Some Applications

Secret Love Letters
Anonymous Crime Tips
Distribution of Music

Sender and receiver anonymity can be achieved with a trusted third party

Sender and receiver anonymity can be achieved with a trusted third party



In This Talk

We will present a scheme for anonymous communication that is efficient and requires no trusted third parties

The Model

Reliable Communication

The adversary can see all communications in network

The adversary can own some of the participants

A participant owned by the adversary may act arbitrarily

The Rest of the Talk

DC Nets
Why DC Nets have never been implemented
k-Anonymity
An efficient scheme

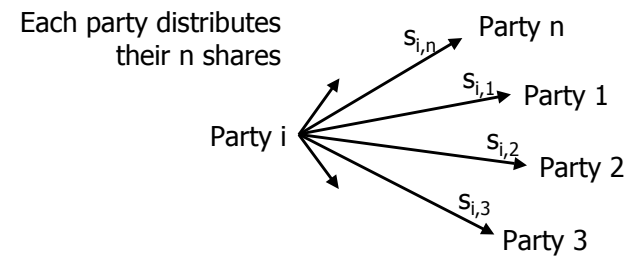
DC Nets: Key Idea

Divide time into small steps
At step t , party i wants to send message $M_i \in Z_m$
If party j doesn't want to send a message at step t , they must send $M_j=0$

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Each party i splits M_i into n random shares
$$M_i = s_{i,1} + s_{i,2} + \dots + s_{i,n-1} + \underbrace{(M_i - (s_{i,1} + \dots + s_{i,n-1}))}_{s_{i,n}}$$

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All parties add up every share that they have received and broadcast the result
(Let B_i denote Party i 's broadcast)

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$$B_i = s_{1,i} + s_{2,i} + \dots + s_{n,i}$$

$$B_1 + B_2 + \dots + B_n = M_1 + M_2 + \dots + M_n$$

DC Nets: Key Idea

If only one of the M_i is nonzero, then:

$$B_1 + B_2 + \dots + B_n = M_i$$

DC Nets: Problems

It is very easy for the adversary to jam the channel!

Communication complexity is $O(n^2)$

Full Anonymity Versus k-Anonymity

We will relax the requirement that the adversary learns nothing about the origin of a given message

We will accept k-anonymity, in which the adversary can only narrow down his search to k participants

The Rest of the Talk

DC Nets

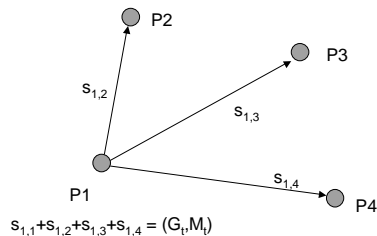
Why DC Nets have never been implemented

k-Anonymity

An efficient scheme

k-anonymous message transmission (k-AMT)

Idea: Divide N parties into "small" DC-Nets of size $O(k)$. Encode M_t as (group, msg) pair



How to compromise k-anonymity

- If everyone follows the protocol, it's *impossible* to compromise the anonymity guarantee.
- So instead, don't follow the protocol: if Alice can never send anonymously, she will have to communicate using onymous means.

How to break k-AMT (I)

- Don't follow the protocol: after receiving shares $s_{1,i}, \dots, s_{k,i}$, instead of broadcasting s_i , generate a random value r and broadcast that instead.
- This will randomize the result of the DC-Net protocol, preventing Alice from transmitting.

Stopping the "randomizing" attack

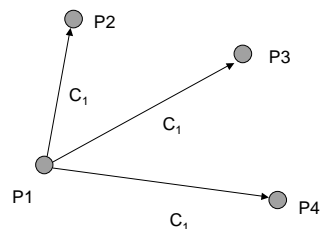
- Solution: Use *Verifiable Secret Sharing*. Every player in the group announces (by broadcast) a commitment to all of the shares of her input.
- These commitments allow verification of her subsequent actions.

k-anonymous message transmission (k-AMT) with VSS

Before starting, each player *commits* to $s_{i,1} \dots s_{i,k}$ via *Pedersen commitment* $C(s,r) = g^s h^r$

$$s_{1,1} + s_{1,2} + s_{1,3} + s_{1,4} = x_1 = (G, M)$$

C	1
1	$g^{s_{1,1}} h^{r_{1,1}}$
2	$g^{s_{1,2}} h^{r_{1,2}}$
3	$g^{s_{1,3}} h^{r_{1,3}}$
4	$g^{s_{1,4}} h^{r_{1,4}}$

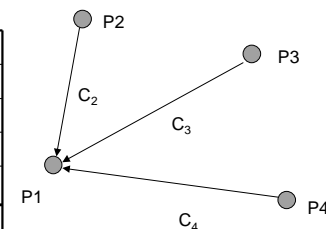


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C	1...	k	
1	$g^{s_{1,1}} h^{r_{1,1}}$	$g^{s_{1,k}} h^{r_{1,k}}$	$g^{s_1} h^r$
2	$g^{s_{2,1}} h^{r_{2,1}}$	$g^{s_{2,k}} h^{r_{2,k}}$	$g^{s_2} h^r$
3	$g^{s_{3,1}} h^{r_{3,1}}$	$g^{s_{3,k}} h^{r_{3,k}}$	$g^{s_3} h^r$
4	$g^{s_{4,1}} h^{r_{4,1}}$	$g^{s_{4,k}} h^{r_{4,k}}$	$g^{s_4} h^r$
	$g^{x_1} h^r$	$g^{x_k} h^r$	



How to break k-AMT (II)

- The multiparty sum protocol gives k participants a single shared channel: at most one person can successfully transmit each turn.
- So: Transmit every turn! VSS still perfectly hides the value of each input; no one will know who is hogging the line.

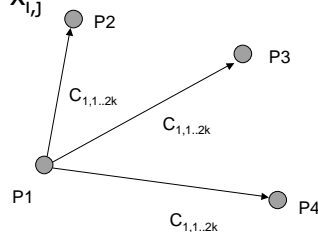
Accommodating more than one sender per turn

- Idea: we can run several turns in parallel. Instead of sending commitments to shares of a single value, generate shares of $2k$ values.
- If Alice picks a random "turn" to transmit in, she should have probability at least $\frac{1}{2}$ of successfully transmitting.

Accommodating more than one sender per turn

Before starting, each player picks slot s , sets $x_{i,s} = (G_t, M_t)$, $x_{i,1} = \dots = x_{i,2k} = 0$, and chooses $s_{i,j,m}$ so that $\sum_m s_{i,j,m} = x_{i,j}$

C	1,1... 1,2k	1,2k
1	$g^{s_{1,1,1}}h^{t_{1,1}}$	$g^{s_{1,2k,1}}h^{t_{1,1}}$
2	$g^{s_{1,1,2}}h^{t_{1,2}}$	$g^{s_{1,2k,2}}h^{t_{1,2}}$
3	$g^{s_{1,1,3}}h^{t_{1,3}}$	$g^{s_{1,2k,3}}h^{t_{1,3}}$
4	$g^{s_{1,1,4}}h^{t_{1,4}}$	$g^{s_{1,2k,4}}h^{t_{1,4}}$
	$g^{x_{1,1}}h^t$	$g^{x_{1,2k}}h^t$



Accommodating more than one sender per turn

- Suppose at the end of the protocol, at least k of the $2k$ parallel turns were empty (zero). Then Alice should be happy; she had probability $\frac{1}{2}$ to transmit.
- If not, somebody has cheated and used at least 2 turns. How do we catch the cheater?

Catching a cheater

- Idea: each party can use her committed values to *prove* (in *zero knowledge*) that she transmitted in at most one slot, without revealing that slot.
- If someone did cheat, she will have a very low probability of convincing the group she did not.

Zero-Knowledge proof of protocol conformance

- $P_i \rightarrow (\text{All})$:
Pick permutation ρ on $\{1 \dots 2k\}$
Send $C(x') = C(x_{\rho(0)}, r'_0), \dots, C(x_{\rho(2k)}, r'_{2k})$
- $(\text{All}) \rightarrow P_i$: $b \in \{0, 1\}$
- $P_i \rightarrow (\text{All})$:
if $b = 0$: open $2k-1$ 0 values;
else reveal ρ , prove (in ZK) $x' = \rho(x)$

Efficiency

- $O(k^2)$ protocol messages to transmit $O(k)$ anonymous messages: $O(k)$ message overhead
- Cheaters are caught with high probability
- Zero Knowledge proofs are *Honest Verifier* and can be done non-interactively in the Random Oracle Model, or interactively via an extra round (commit to verifier coins)